School of Mathematical and Computational Sciences

Indian Association for the Cultivation of Science

Master's/Integrated Master's-PhD Program/ Integrated Bachelor's-Master's Program/PhD Course

Theory of Computation II: COM 5108

Tutorial X (23 November 2023)

Instructor: Goutam Biswas *Autumn Semester 2023*

1. Prove that UNSAT = $\{\phi$: the boolean formula ϕ is not satisfiable $\} \in$ **AP**.
2. NOT-EQIV = $\{<\phi, \psi>$: boolean formulas ϕ and ψ are not equivalent$\}$. Prove that NOT-EQUIV is in **NP**.
3. $NONMIN - FORMULA = \{<\phi>$: ϕ is not a minimal formula$\}$. Prove that NONMIN-FORMULA is in $\mathbf{NP}^{SAT}$.
4. $\phi(x_1, \cdots, x_n)$ is a CNF formula with c clauses and n variables. Design an NFA N so that

$$L(N) = \{v \in \{0,1\}^n : \ v = v_1 \cdots v_n \text{ and } \phi(v_1, \cdots, v_n) = \ \textit{false}\}.$$

5. The language class **DP** is defined as follows:

$$\mathbf{DP} = \{L = L_1 \cap L_2 : \ L_1 \in \mathbf{NP}, L_2 \in \mathbf{coNP}\}.$$

   (a) Is $\mathbf{NP} \cap \mathbf{coNP} = \mathbf{DP}$?

   (b) Show that
   EXACT-INDSET = $\{<G, k>$: the size of the largest independent set of G is $k\} \in$ **DP**.

   (c) Prove that $\mathbf{NP} \cup \mathbf{coNP} \subseteq \mathbf{DP}$.