School of Mathematical and Computational Sciences

Indian Association for the Cultivation of Science

Master's/Integrated Master's-PhD Program/ Integrated Bachelor's-Master's Program/PhD Course

Theory of Computation II: COM 5108

Tutorial I (10 August 2023)

Instructor: Goutam Biswas *Autumn Semester 2023*

1. (a) Design an NFA with minimum number of states that accepts all strings over $\{0, 1\}$ which has "101" as a substring.

   (b) Convert the NFA to a DFA by subset construction (do not design it directly).

   (c) How many states are there in the DFA that are reachable from the start state?

   (d) Can we minimize the reachable part of the DFA designed in (**1c**)?

2. (a) How do you identify the states that are not reachable from the start state?

   (b) What is the time complexity of your algorithm?

3. Given a DFA $M = (Q, \Sigma, \delta, s, F)$ we define a binary relation $\equiv$ on the set of states Q as follows:
   For all $p, q \in Q$, $p \equiv q$ if $\forall x \in \Sigma^* \ (\delta(p, x) \in F \Leftrightarrow \delta(q, x) \in F)$.

   (a) Justify that it is an *equivalence relation*.

   (b) If $p \equiv q$, define two automaton with Q, Σ, δ, F of M whose languages are same.

   (c) The *equivalence relation* of (**3**) partitions the set of states. Define the *quotient automaton*.

   (d) How is $L(M)$ related to the language of the *quotient automaton*?

4. Let $L \subseteq \Sigma^*$ be a regular language accepted by a DFA, $M = (Q, \Sigma, \delta, s, F)$ without any inaccessible state from the start state.
   Define a binary relation $\equiv_M$ (modulo M) on Σ^* as follows:

   $$x \equiv_M y \ \Leftrightarrow \ \delta(s, x) = \delta(s, y).$$

   Two strings are related if they drive the machine from its start state to the same state.

   (a) Show that $\equiv_M$ is an *equivalence relation*.

   (b) How many equivalence classes of Σ^* are created by $\equiv_M$? (*finite index*)

   (c) Let $x \equiv_M y$ and $a \in \Sigma$. What can you conclude about xa and ya? (*right congruence*).

   (d) If $x \in L$, then what can you conclude about $[x]_{\equiv_M}$?

(e) How do you express L in terms of the equivalences classes of $\equiv_M$?

5. An equivalence relation over Σ^* for a language L is called a *Myhill-Nerode relation* for L, if
(i) it satisfies *right congruence*,
(ii) it *refines* L, and
(iii) it has *finite index*.

   (a) Given any *Myhill-Nerode relation* $\equiv_{mn}$ on Σ^* for L, construct a DFA that accepts L. Let $M_{mn} = (Q, \Sigma, \delta, s, F)$, where

   (b) Prove that $\delta([x], y) = [xy]$.

   (c) Prove that $L = L(M_{mn})$.