School of Mathematical and Computational Sciences
Indian Association for the Cultivation of Science

Master's/Integrated Master's-PhD Program/ Integrated Bachelor's-Master's Program/PhD Course

Theory of Computation II: COM 5108

Quiz III (16 November 2023)

Answer All Questions with Proper Justifications

Marks: 6 × 3 = 18

1. What is the maximum possible number of configurations of an $f(n) \geq \log n$ space bounded (work tape) TM, where n is the input length.

2. Let $\phi(C_1, C_2, t)$ be a TQBF formula corresponding to a $O(n^k)$ space bounded computation of a TM going from configuration C_1 to configuration C_2 in t steps. An intermediate configuration C_m is such that both C_1 to C_m and C_m to C_2 are reached in $\frac{t}{2}$ steps. So we can write

$$\phi(C_1, C_2, t) = \exists C_m(\phi(C_1, C_m, t/2) \wedge \phi(C_m, C_2, t/2)).$$

   Explain why this scheme cannot give a polynomial size formula.

3. Let $A = \{x \in \{0,1\}^* :$ number of 0's in x is twice the number of 1's in it$\}$. Is $A \in \mathbf{L}$?

4. We define the language

$$H_f = \{< M, x >: \text{ the DTM } M \text{ accepts the input } x \text{ in } f(|x|) \text{ steps}\}.$$

   We claim that $H_f \notin \mathbf{DTIME}(f(\lfloor \frac{n}{2} \rfloor))$. We prove this by *diagonalization*. Suppose the TM M_f decides H_f within $f(\lfloor \frac{n}{2} \rfloor)$ number of steps. We construct the following TM D_f for diagonalization.

   D_f: input: $< M >$
   Simulate N_f on $< M, M >$.
   if N_f *accepts*, then **reject** else **accept**.

   (a) Find the running time of D_f on input of length $n = | < M > |$.

   (b) Apply D_f on its own description $< D_f >$ and give the argument for contradiction.

5. $A_{cof} = \{< M >: \overline{L(M)} = \Sigma^* \setminus L(M) \text{ is finite}\} \in \Sigma_n^0$. What is the least value of n? Give a definition of A_{cof} based on a recursive predicate.

6. People claim that $\mathbf{PH} = \bigcup_{n \geq 0} \Sigma_n^p$ cannot have a complete language. If L is a $\mathbf{PH}$-complete language it must belong to Σ_i^p for some i.
   Explain what can be the conclusion in that case?