



School of Mathematical and Computational Sciences
Indian Association for the Cultivation of Science

*Master's/Integrated Master's-PhD Program/ Integrated
Bachelor's-Master's Program/PhD Course*

Theory of Computation II: COM 5108

Quiz II (12 October 2023)

Answer All Questions

Marks: (1+2) + (1+1) + 4 + (4 + 2) = 15

1. A language L is **PSPACE**-complete.

- (i) Is L **NP**-hard?
- (ii) Is L **NP**-complete?

Justify your answers and assumption.

2. Does ϕ and/or ψ belong to TQBF?

- (i) $\psi = \exists b \forall a \exists c (a \vee b) \wedge (\bar{a} \vee c) \wedge (\bar{b} \vee \bar{c})$.
- (ii) $\phi = \forall a \exists b \exists c (a \vee b) \wedge (\bar{a} \vee c) \wedge (\bar{b} \vee \bar{c})$.

3. Consider the CFG $G = (\{0, 1\}, \{S\}, R, S)$, where the production rules are $S \rightarrow \varepsilon \mid 0S1 \mid SS$.

Is $L(G)$, the language of G , in **L**?
Clearly justify your answer.

4. A GG game is a 2-player ($\{I, II\}$) game played as follows. G is a directed graph with a designated start node s .

- (i) The player ' I ' starts the game from the start node s . Each player gives alternate moves.
- (ii) A move by a player is to pick a new node in the graph on a *simple directed path* from the current node. A *simple path* is one where no node has already been visited.
- (iii) A player loses if she fails to make a move.

$GG = \{ \langle G, s \rangle : \text{player 'I' has a winning strategy on the directed graph } G, \text{ starting from } s \}$.

- (a) Following algorithm (incomplete) decides GG. **Fill-in the blanks** to complete the algorithm. Give proper justifications.

M : Input $\langle G, s \rangle$

- 1 If *out-degree* of s is zero, then $\dots(i)\dots$ -halt
- 2 Remove s and all edges in and out of it. The new graph is G' where s_1, s_2, \dots, s_k are nodes pointed by s in G .
- 3 Give recursive calls to M with parameters $\dots(ii)\dots$.
- 4 If all these calls reach '**Yes**'-halt, then $\dots(iii)\dots$ - halt.
- 5 Otherwise, $\dots(iv)\dots$ - halt.

- (b) Justify that $GG \in \mathbf{PSPACE}$.