School of Mathematical and Computational Sciences

Indian Association for the Cultivation of Science

Master's/Integrated Master's-PhD Program/ Integrated Bachelor's-Master's Program/PhD Course

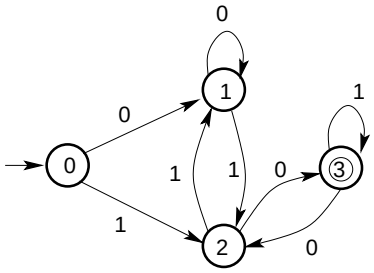
Theory of Computation II: COM 5108

Quiz I (31 August 2023)

Answer All Questions

Marks: 5 × 2 = 10

1. Consider the following DFA and characterize the equivalence classes over $\{0,1\}^*$ induced by its states (right congruence equivalence relation).



2. $f, g : \mathbb{N} \to \mathbb{N}$, $f(n) = O(n^2)$ and $g(n) = O(n)$. What is tight upper-bound of $g \circ f$ (g composition f)?

3. Let $\mathscr{P}\mathbb{N}_{fin}$ be the collection of finite subsets of $\mathbb{N} = \{0, 1, 2, \cdots\}$. We define $f : \mathscr{P}\mathbb{N}_{fin} \to \mathbb{N}$ as $f(\{a_1, \cdots, a_k\}) = \sum_{i=1}^{k} 2^{a_i}$.
What type of mapping is this (one-one, onto, both)? What can you conclude about the size of $\mathscr{P}\mathbb{N}_{fin}$?

4. Let $L = \{< M, x >:$ M is a DFA and M rejects $x\}$. Is L a *decidable* language?

5. Let L_1 and L_2 be two *recursively enumerable (Turing recognizable)* languages recognized by DTM M_1 and M_2 respectively. Two DTM $M_\cup$ and $M_\cap$ are designed using M_1 and M_2, to recognized $L_1 \cup L_2$ and $L_1 \cap L_2$ respectively.
Which one of $M_\cup$ and $M_\cap$ should simulate M_1 and M_2 in parallel on two copies of the input? Justify your answer.