Indian Association for the Cultivation of Science

(Deemed to be University under *de novo* Category)

Master's/Integrated Master's-PhD Program/ Integrated Bachelor's-Master's Program/PhD Course

Mid-Semester (Sem-III) Examination-Autumn 2023

Subject: **Theory of Computation II** *Subject Code:* **COM 5108‘**

Full Marks: **25** *Time Allotted:* **2 hours**

Q 1. Answer five (5) questions with brief justifications. [5 × 2 = 10]

(a) Is $2^n = o(3^n)$ (small 'o')?

(b) Draw the implication graph (directed) corresponding to the 2SAT formula $\phi = (x \vee \overline{y}) \wedge (y \vee \overline{z}) \wedge (z \vee \overline{x}) \wedge (\overline{z} \vee \overline{x})$. Justify from the graph why all variables get the truth value *false* for a *satisfying* assignment.

(c) An NTM has a 3-way nondeterminism at the given state-input pair: $\delta(q_0, 0) = \{(q_1, 0, \leftarrow), (q_2, 0, -), (q_3, 1, \rightarrow)\}$. Modify the transition function so that there will be 2-way nondeterminism.

(d) The language $coFin = \{L \subseteq \Sigma^* : \Sigma^* \setminus L \text{ is finite}\}$. Is $coFin \in \mathbf{P}$?

Error. it is not '$\in$' but '$\subseteq$'.

(e) Is the following language in **P** or **NP**-hard?

$$SAT_e = \{\phi\ 0\ 1^{2^n} : \phi \in SAT \text{ and } |\phi| = n\}.$$

(f) What is the total number of variable instances used in this formula where $C = Q \cup \Gamma$ and $|C| = l$.

$$\phi_{cell} = \bigwedge_{1 \le i,j \le n^k} \left(\left(\bigvee_{p \in C} x_{i,j,p} \right) \wedge \left(\bigwedge_{\substack{p,q \in C, \\ p \neq q}} (\overline{x_{i,j,p}} \vee \overline{x_{i,j,q}}) \right) \right).$$

(g) A language $L \in \mathbf{NP}$ is defined as follows.

$\forall x \in \Sigma^*$, $x \in L$ if and only if $\exists w \in \Sigma^{p(n)}$, there is a $q(n)$ time bounded DTM V that accepts $<x, w>$,

where $n = |x|$, $p(n)$, $q(n)$ are polynomials.
Define a language $L' \in \mathbf{coNP}$ in a similar way.

Answer any three (3) of the following questions.

[3 × 5 = 15]

Q 2. Give an $O(n \log n)$ step bounded single-tape DTM algorithm to recognize $L = \{x \in \{0,1\}^* : x$ has equal number of 0's and 1's$\}$, where n is the length of the input. Detail state transitions are not required, but explain its operation and the time complexity.

Q 3. Give the detail design of a DTM $M = (Q, \Sigma, \delta, s)$, that takes the input $\rhd x$, where $x \in \{0,1\}^+$ and computes the 2's complement of x (ignore overflow). Examples of computation are: $\rhd 0 \rightarrow_M^* \rhd 0$, $\rhd 1 \rightarrow_M^* \rhd 0$, $\rhd 101100 \rightarrow_M^* \rhd 010100$.

Q 4. $3COLOR = \{< G >: G$ is an undirected graph whose vertices can be coloured with at most 3 colours $\}$. Give a polynomial time reduction of $3COLOR$ to SAT, $G \mapsto \phi$, G is *3-colourable* if and only if ϕ is *satisfiable*.

Q 5. Informally describe how an $f(n)$ time bounded k-tape DTM can be simulated on a single-tape DTM. What is the order of *slowdown* of the simulation?