Indian Association for the Cultivation of Science
(Deemed to be University under *de novo* Category)

Master's/Integrated Master's-PhD Program/ Integrated Bachelor's-Master's Program/PhD Course

End-Semester (Sem-III) Examination-Autumn 2023

Subject: **Theory of Computation II** *Subject Code:* **COM 5108**

Full Marks: **50** *Time Allotted:* **3 hours**

Q 1. Answer ten(10) questions with brief justifications. [**10** $\times$ **2** = **20**]

(a) Is $DTIME(2^n)$ a proper subset of $DTIME(2^{2n})$?

(b) What is the largest carry generated while multiplying (school-book method) two n-bit positive integers? Can it be computed in *logspace*?

(c) Give a *context sensitive* language A such that $A \in \mathbf{L}$.

(d) Is the language $L = \{< \phi, \psi >: \phi \in SAT \wedge \psi \in UNSAT\}$ **NP**-hard?

(e) Let $L = \{x0\phi : \ x \in \{1\}^*, \phi \in SAT\}$. Is $L \in \mathbf{NP}$? Is it possible that $L \in \mathbf{P}$ for a suitable choice of x?

(f) Why do people believe that **PH** cannot have a complete problem?

(g) Can you justify from the cardinality (size of set) argument that there are too many languages in **P/poly** and not all of them can be computable.

(h) Prove that, if $\Sigma_3^P \subseteq \Pi_3^P$, then $\Sigma_3^P = \Pi_3^P$.

(i) a, b, c are Boolean variables. Express $(a \wedge b) = c$ in 3CNF form.

(j) Define the probablistic language class **RP**.

(k) What is the basic difference between the class **BPP** and **ZPP**?

(l) Assume that multiplication ($Mult(m, n)$) is defined. Give a *primitive recursive* definition of n^m ($Exp(m, n)$).

(m) Let the encodings of *True* $\equiv \lambda xy \cdot x$; *False* $\equiv \lambda xy \cdot y$; *pair* $\equiv \lambda xyz \cdot (zxy)$; *second* $\equiv \lambda x \cdot (x(\lambda yz \cdot z))$, as λ-terms.
Evaluate: *second (pair A B)*, where A and B are λ terms.

(n) Is $TQBF = \{\phi : \ \phi$ is a closed, *true* QBF$\}$ a complete problem of **AP**?.

Answer any three (3) of the following questions.

[3 × 10 = 30]

Q 2. **[6+4]**

(a) Explain why PATH = $\{< G, s, d >: G$ is a directed graph with a path from s to $d\ \}$ is **NL**-hard.

(b) Justify that 2SAT is in **NL**.

Q 3. **[5+5]**

(a) Give the outline of a proof that there is an oracle set A such that $\mathbf{P}^A = \mathbf{NP}^B$. [**Error.** corrected in the exam hall: $\mathbf{NP}^A$].

(b) Let $MAX - INDSET = \{< G, k >:$ the size of the largest independent set of G is $k\ \}$. Show that $MAX - INDSET \in \mathbf{P}^{SAT}$.

Q 4. **[2+5+3]**

(a) CKT-SAT = $\{< C >: C$ is a satisfiable Boolean Circuit$\}$. Show that CKT-SAT is in **NP**.

(b) How do you reduce CKT-SAT to 3SAT?

(c) Let $A \in \mathbf{P}$. Show that there is a **P**-uniform circuit family for A.
Assume that for a language $A \in DTIME(f(n))$, $f(n) \geq n$, a circuit of size $O(f(n)^2)$ can be constructed by an $O(f(n)^2)$ time bounded TM.

Q 5. **[5+5]**

(a) Justify that $ATIME(f(n)) \subseteq DSPACE(f(n))$, where $f(n) \geq n$.

(b) Show that **ZPP** $\subseteq$ **RP**.

Q 6. **[5+5]**

(a) How do you compute $g \circ f$ in logspace, if $f, g : \{0,1\}^* \to \{0,1\}^*$ are computable in logspace?

(b) Justify that $TQBF = \{\phi :\ \phi$ is a closed, *true* QBF$\}$ is in **PSPACE**.

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