School of Mathematical and Computational Sciences

Indian Association for the Cultivation of Science

Master's/Integrated Master's-PhD Program/ Integrated Bachelor's-Master's Program/PhD Course

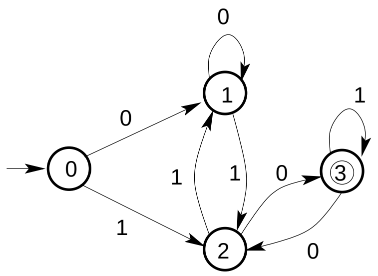
Theory of Computation II: COM 5108

Quiz I (31 August 2023)

Answer All Questions

Marks: 5 × 2 = 10

1. Consider the following DFA and characterize the equivalence classes over $\{0,1\}^*$ induced by its states (right congruence equivalence relation).



Ans. $x \neq \varepsilon$ are treated as binary numeral.

(a) State 0: $\{\varepsilon\}$.

(b) State 1: $\{x : \ x \equiv 0(\text{mod}3)\}$.

(c) State 2: $\{x : \ x \equiv 1(\text{mod}3)\}$.

(d) State 3: $\{x : \ x \equiv 2(\text{mod}3)\}$.

2. $f, g : \mathbb{N} \to \mathbb{N}$, $f(n) = O(n^2)$ and $g(n) = O(n)$. What is tight upper-bound of $g \circ f$ (g composition f)?

Ans. $O(n^2)$.

3. Let $\mathscr{P}\mathbb{N}_{fin}$ be the collection of finite subsets of $\mathbb{N} = \{0, 1, 2, \cdots\}$. We define $f : \mathscr{P}\mathbb{N}_{fin} \to \mathbb{N}$ as $f(\{a_1, \cdots, a_k\}) = \sum_{i=1}^{k} 2^{a_i}$.
What type of mapping is this (one-one, onto, both)? What can you conclude about the size of $\mathscr{P}\mathbb{N}_{fin}$?

Ans. The mapping is *one-one* through the binary representation of the elements of $\mathbb{N}$. The size of $\mathscr{P}\mathbb{N}_{fin}$ is *countably infinite* as there is an *one-one* map $g : \mathbb{N} \to \mathscr{P}\mathbb{N}_{fin}$, $g(n) = \{n\}$.

4. Let $L = \{< M, x >$: M is a DFA and M rejects $x\}$. Is L a *decidable* language?

   Ans. Let N be a DTM with 3-tapes. The input tape is read-only and there are two work tapes.
   N checks whether $< M >$ is a valid description of a DFA. If it is not, it may simply *halt*s with 'N'.
   If $< M >$ is a valid description, N copies x on the 2nd tape and the *start* state of the DFA to the 3rd tape. The 3rd tape always contains the current state of the DFA.
   The DTM (i) reads the current state and current symbol from tape 3 and 2,
   (ii) consults the transition table of the machine M from the 1st tape,
   (iii) writes the next state on the 3rd tape, moves the head on input.
   When the input is '$\sqcup$', N halts with 'Y' if state on tape 3 is non-final (of DFA M), otherwise it halts with 'N'.

5. Let L_1 and L_2 be two *recursively enumerable (Turing recognizable)* languages recognized by DTM M_1 and M_2 respectively. Two DTM $M_\cup$ and $M_\cap$ are designed using M_1 and M_2, to recognized $L_1 \cup L_2$ and $L_1 \cap L_2$ respectively.
   Which one of $M_\cup$ and $M_\cap$ should simulate M_1 and M_2 in parallel on two copies of the input? Justify your answer.

   Ans. The DTM $M_\cup$ should simulate M_1 and M_2 in parallel. Let the input to $M_\cup$ be x and it first simulates M_1 on x. If $x \notin L_1$, the simulation may not terminate. But x may belong to L_2.