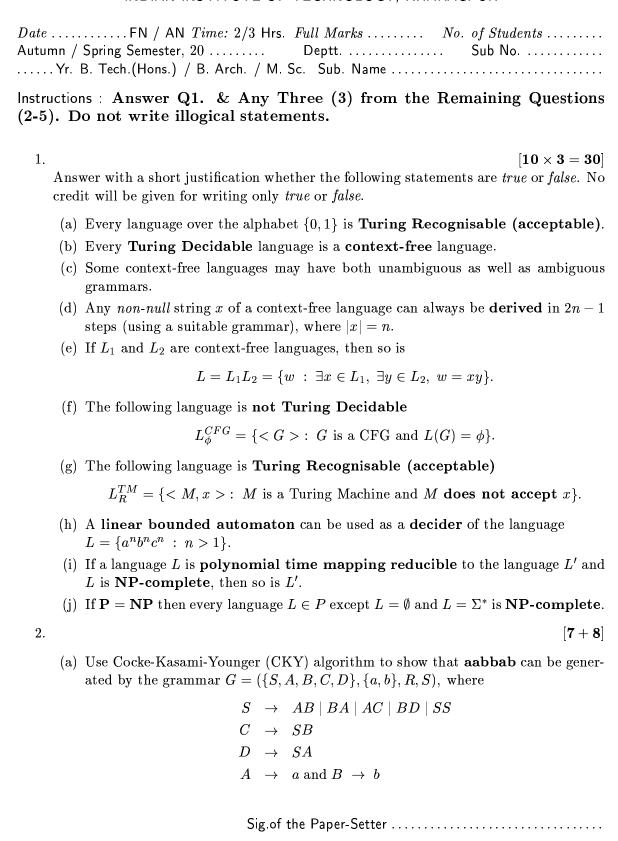
## INDIAN INSTITUTE OF TECHNOLOGY, KHARAGPUR



| (b) | Give a context-free grammar for the following language and convert the grammar   |
|-----|--|
|     | to Chomsky Normal Form: $\mathbf{L} = \{\mathbf{a^k b^m c^n} : \mathbf{k}, \mathbf{m} \ge 1, \ \mathbf{k} \le \mathbf{n} \le 2\mathbf{k}\}.$ |

3. [10+5]

- (a) Design a single tape deterministic Turing machine  $M = (Q, \Sigma, \Gamma, \delta, q_0, q_A, q_R)$  that accepts the following language in  $\mathbf{O}(\mathbf{n} \log \mathbf{n})$  time :  $L = \{a^n b^n : n \geq 0\}$ . Clearly specify each component of M and give the state transition function  $\delta$  as a state transition diagram (give short explanation for each transition). Assume that there is a special marker  $\mathcal{S} \not\in \Sigma$  at the leftmost square of the tape.
- (b) Give the outline of proof of the following statement:  $L = \{ < G > : G \text{ is a CFG over } \{0,1\} \text{ and } \{1\}^* \cap L(G) \neq \emptyset \}$  is a **decidable language**.

4. [3+7+5]

- (a) Prove that the following language is **Turing Recognisable (acceptable)**:  $L_{HALT}^{TM} = \{ \langle M, x \rangle : M \text{ is a Turing machine and } M \text{ halts on } x \}.$
- (b) Use diagonalisation and reduction to a contradiction (not the reduction of another language to  $L_{HALT}^{TM}$ ) to prove that the language L of (4a) is undecidable.
- (c) Let  $L_1$  and  $L_2$  be two disjoint languages. Say that language L separates  $L_1$  and  $L_2$  if  $L_1 \subseteq L$  and  $L_2 \subseteq \overline{L}$ . Show that two disjoint co-Turing-Recognisable languages are separated by some decidable language.

5. [4+7+4]

- (a) Give justification for the following statement:

  All deterministic Turing machine models are polynomially equivalent.
- (b) Prove that the following language

 $TRIANGLE = \{ \langle G \rangle : G \text{ is a undirected graph and } G \text{ has a 3-clique} \},$ 

is in the complexity class **P**. A 3-clique is a complete subgraph of three vertices.

(c) **3SAT** is known to be an NP-complete language. Show by polynomial time mapping reduction that **VERTEX-COVER** is also an NP-complete language.

\*\*\* END \*\*\*

Sig.of the Paper-Setter .....