Computer Science & Engineering Department
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Foundations of Computing: CS30053
3rd Year : Autumn Semester
Class Test II (**Answers**)

From 1730hr to 1830hr *Date : 28th October, 2003*

1. Justify that a finte language L (there are finite number of strings in L) is always a *context-free* language. [**2**]

   Ans. Let $L = \{x_1, x_2, \cdots, x_n\}$. The CFG is simply $G = (\{S\}, \Sigma, R, S)$, where R is

$$S \rightarrow x_1 \mid \cdots \mid x_n.$$

2. Consider the context-free grammar $G = (\{A, B\}, \{0, 1\}, R, A)$, where

$$R = \{A \rightarrow BB,\ B \rightarrow BB,\ B \rightarrow 0,\ B \rightarrow 1B,\ B \rightarrow B1\}.$$

[**2 + 2 + 1**]

   (a) List all strings of $L(G)$ produced in **three (3)** or fewer steps of derivations. **Ans.** At each step of derivation exactly one nonterminal will be replaced by the right side of a production rule.

Step - I	**Step - I**	**Step - III**
$A \Rightarrow BB$	$\Rightarrow BBB$	
	$0B$	$\Rightarrow 00$
	$1BB$	
	$B1B$	
	$B0$	$\Rightarrow 00$
	$BB1$	

   (b) Give two parse trees of derivations corresponding to the string '**0010**'.

   Ans. Two such parse trees are corresponding to the following two leftmost derivations.

   - $A \Rightarrow BB \Rightarrow BBB \Rightarrow 0BB \Rightarrow 00B \Rightarrow 001B \Rightarrow 0010.$
   - $A \Rightarrow BB \Rightarrow BBB \Rightarrow 0BB \Rightarrow 0B1B \Rightarrow 001B \Rightarrow 0010.$

   (c) What is your conclusion about the grammar?

   Ans. The grammar is ambiguous.

3. Give context-free grammars for the following languages. Write short explanation about the production rules. [**5 + 5**]

(a) $L_1 = \{a^m b^n c^p d^q \ : \ m+n = p+q\}$.
Ans. One such grammar is $G = (\{S, B, D, E\}, \{a, b, c, d\}, R, S)$, where R is

$$\begin{aligned} S &\rightarrow aSd \mid B \mid D \\ B &\rightarrow bBd \mid E \\ D &\rightarrow aDc \mid E \\ E &\rightarrow bEc \mid \varepsilon \end{aligned}$$

(b) $L_2 = \{uaxb \ : \ u, x \in \{0,1\}^*, \ |u| = |x|\}$.
Ans. One such grammar is $G = (\{S, A, T\}, \{a, b, 0, 1\}, R, S)$, where R is

$$\begin{aligned} S &\rightarrow Ab \\ A &\rightarrow TAT \mid a \\ T &\rightarrow 0 \mid 1 \end{aligned}$$

4. Let Σ and Γ be two alphabets and

$$h : \Sigma \longrightarrow \Gamma^*$$

be a map so that **each element** of Σ is mapped to a **string** over Γ. Let L be a language over Σ.

$$h(L) = \{y \in \Gamma^* \ : \ \exists x \in L, \text{so that } x = \sigma_1 \cdots \sigma_n \text{ and } y = h(\sigma_1) \cdots h(\sigma_n)\}.$$

Each $\sigma_i \in \Sigma$, $1 \leq i \leq n$. The value of $h(\varepsilon) = \varepsilon$.

Justify that $h(L)$ is a context-free language if L is context-free. [**3**]

Ans. Replace every terminal σ in the right side of every production rule by $h(\sigma)$. The grammar with the modified rules and the same set of non-terminals and the start symbol is the grammar for $h(L)$.