

**Computer Science & Engineering Department**  
**I. I. T. Kharagpur**

**Foundations of Computing: CS30053**

*3rd Year : Autumn Semester*

*Class Test II (Total Marks: 20)*

From 1730hr to 1830hr

*Date : 28th October, 2003*

**Answer All Questions**  
**Do not write *illogical* statements.**

1. Justify that a finite language  $L$  (there are finite number of strings in  $L$ ) is always a *context-free* language. [2]

2. Consider the context-free grammar  $G = (\{A, B\}, \{0, 1\}, R, A)$ , where

$$R = \{A \rightarrow BB, B \rightarrow BB, B \rightarrow 0, B \rightarrow 1B, B \rightarrow B1\}.$$

[2 + 2 + 1]

(a) List all strings of  $L(G)$  produced in **three (3)** or fewer steps of derivations.

(b) Give two parse trees of derivations corresponding to the string '0010'.

(c) What is your conclusion about the grammar?

3. Give context-free grammars for the following languages. Write short explanation about the production rules. [5 + 5]

(a)  $L_1 = \{a^m b^n c^p d^q : m + n = p + q\}$ .

(b)  $L_2 = \{uaxb : u, x \in \{0, 1\}^*, |u| = |x|\}$ .

4. Let  $\Sigma$  and  $\Gamma$  be two alphabets and

$$h : \Sigma \longrightarrow \Gamma^*$$

be a map so that **each element** of  $\Sigma$  is mapped to a **string** over  $\Gamma$ . Let  $L$  be a language over  $\Sigma$ .

$$h(L) = \{y \in \Gamma^* : \exists x \in L, \text{ so that } x = \sigma_1 \cdots \sigma_n \text{ and } y = h(\sigma_1) \cdots h(\sigma_n)\}.$$

Each  $\sigma_i \in \Sigma$ ,  $1 \leq i \leq n$ . The value of  $h(\varepsilon) = \varepsilon$ .

Justify that  $h(L)$  is a context-free language if  $L$  is context-free. [3]