Computer Science & Engineering Department
I. I. T. Kharagpur

Foundations of Computing: CS30053

3rd Year : Autumn Semester
Class Test I (**Total Marks: 20**)

From 1730hr to 1830hr | *Date : 26th August, 2003*

Answer All Questions
Do not write *illogical* statements.

1. Give a *bijection* (in closed form) from the set of natural numbers, $\mathbb{N} = \{0, 1, 2, \cdots\}$ to the set of integers divisible by **5**, $\mathbb{Z}_5 = \{\cdots, -10, -5, 0, 5, 10, \cdots\}$. [**2**]

2. Let $f : A \longrightarrow \mathbb{N}$ and $g : \mathbb{N} \times \mathbb{N} \longrightarrow C$ be *bijections*. Show that there is a bijection from A to C. [Do not use *Schröder-Bernstein* theorem.] [**3**]

3. Let $f : A \longrightarrow B$ be an *injection* and $g : B \longrightarrow A$ be a *surjection*. Justify or refute -
   - '$\mathbf{g} \circ \mathbf{f}$ is always a **bijection**.'
   - 'if $g(f(A)) = A$, then $\mathbf{g} \circ \mathbf{f}$ is a **bijection**.' [**3**]

4. A *binary relation* R over a set A with n elements ($|A| = n$) is called *reflexive* if $(\mathbf{a}, \mathbf{a}) \in \mathbf{R}$, **for all** $a \in A$. How many reflexive relations are possible on A. Justify your answer. [**3**]

5. Show that there is a *bijection* from the *closed interval* $[0, 1]$ to the *closed interval* $[0, 2]$ on the real line. [**3**]

6. Let V be a *denumerable* set of *variable names*. We inductively define the set of *propositional terms* (P) in the following way.
   - '**0**' and '**1**' are in P.
   - Every $v \in V$ (*variable name*) is in P.
   - If p and q are in P, then so are $(p \Rightarrow q)$ and $\neg p$.
   - Nothing else is in P.

   Justify that P is *denumerable*. [**3**]

7. Consider the alphabet $\Sigma = \{0, 1, *\}$. The set Σ^n is the collection of all strings over Σ of length exactly n i.e. $\Sigma^0 = \{\varepsilon\}$, $\Sigma^1 = \{0, 1, *\}$, $\Sigma^2 = \{\not0, \not1, \not*, 00, 01, 0*, 10, 11, 1*, *0, *1, **\}$, etc. Consider the string

   $$0101010101010101010101010101010$$

   We may describe it as $01*1111$; which may be interpreted as '**01**' repeated '**1111**' times. The description is shorter than the actual string.

   Give a proof that every string of Σ^n cannot have a **shorter description** using the symbols of Σ. [**3**]