Formal Language and Automata Theory (CS21004)

Tutorial - VI

Class: CSE 2^{nd} Year *Date*: 15^{th} February, 2010

Exercise 1. $L = \{x \in \{a,b\}^* : x = a^m b^n,\ n \geqslant m \geqslant 0\}$ is not regular.

Exercise 2. $L = \{x \in \{a,b\}^* : x = a^m b^n,\ m \neq n\}$ is not regular.

Exercise 3. $L = \{x \in \{a,b\}^* : |x|_a \neq |x|_b\}$ is not regular.

Exercise 4. $L = \{x \in \{a,b\}^* : x = a^m b^n,\ m \geqslant n \geqslant 0\}$ is not regular.

Exercise 5. $L = \{x \in \{a,b\}^* : |x|_a > |x|_b\}$ is not regular.

Exercise 6. $L = \{x \in \{a,b\}^* : x = ww, \text{where } w \in \{a,b\}^*\}$ is not regular.

Exercise 7. $L = \{x \in \{a,b\}^* : x = x^R\}$ is not regular.

Exercise 8. $L = \{x \in \{a,b\}^* : x \neq x^R\}$ is not regular.

Exercise 9. $L = \{x \in \{a,b\}^* : x = a^m b^n,\ m + n = l^2, m, n, l \geqslant 0\}$ is not regular.

Exercise 10. $L = \{x \in \{a,b\}^* : x = a^{n!},\ n \geqslant 0\}$ is not regular.

Exercise 11. $L = \{x \in \{a\}^* : x = a^p,\ p \text{ is a prime number}\}$ is not regular.

Exercise 12. $L = \{x \in \{a,b\}^* : |x|_a \text{ is a prime number}\}$ is not regular

Exercise 13. $L = \{x \in \{a,b\}^* : x = a^m b^n,\ \text{GCD}(m,n) \neq 1\}$ is not regular.

Exercise 14. $L = \{x \in \{a,b\}^* : x = a^m b^n,\ \text{GCD}(m,n) = 1\}$ is not regular.

Exercise 15. Let $\Sigma = \{0,\ 1,\ +\ ,\ =\ \}$ and the language $L = \{x\ =\ y\ +\ z\colon x,\ y,\ z \in \{0, 1\}^* \text{ so that } x \text{ is the sum of } y \text{ and } z\}$. L is not regular.

Exercise 16. How do you simulate an NFA in a C program?

Exercise 17. Let $\Sigma = \left\{ \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right\} = \{a_0, a_1, a_2, a_3\}$. A string over Σ may be interpreted as two rows of 0's and 1's e.g. $x = a_0\ a_2\ a_2\ a_3\ a_1\ = \begin{bmatrix} 01110 \\ 00011 \end{bmatrix} = \begin{bmatrix} x_t \\ x_b \end{bmatrix}$. We define the language L over Σ, $L = \{x \in \Sigma^* : 3x_t = x_b\}$. Show that L is regular.

Exercise 18. Let L be a language over Σ. We define the language NOEXTEND$(L) = \{x \in \Sigma^* : x \text{ is not a proper prefix of any string of } L\}$. Show that if L is regular, then so is NOEXTEND(L).

Exercise 19. Let L be a language over $\Sigma = \{a,b\}$. We define
$L_{\frac{1+3}{3}} = \left\{xz \in \{a,b\}^* : \exists y \in \{a,b\}^*,\ |x| = |y| = |z\rfloor \text{ and } xyz \in L\right\}$. The language $L_{\frac{1+3}{3}}$ need not be regular even if L is not regular.

Exercise 20. The *shuffle* of two strings $x, y \in \Sigma^*$ is the set of all strings generated by shuffling the characters of x and y (the relative order of characters in the original strings are preserved). As an example

$ab \parallel cb = \{abcd, acbb, cabb, cbab\}$. We can define $x \parallel y$ inductively as follows:

$$x \parallel y = \begin{cases} x & \text{if } y = \varepsilon, \\ y & \text{if } x = \varepsilon, \\ (u \parallel vb) \cdot \{a\} \cup (ua \parallel v) \cdot \{b\}, & \text{if } x = ua, y = vb. \end{cases}$$

The *shuffle* of two languages L_1 and L_2 is $L_1 \parallel L_2 = \bigcup_{x \in L_1, y \in L_2} x \parallel y$

As an example $\{ab, a\} \| \{cb\} = \{ abcb, acbb, cabb, cbab, acb, cab, cba \}$.
Show that if L_1 and L_2 are regular, then $L_1 \| L_2$ is also regular.

Exercise 21. If L is a language over Σ, let $L_{\frac{1}{2}} = \{x \in \Sigma^* : \text{there is } y \in \Sigma^*,\ xy \in L\}$. Show that $L_{\frac{1}{2}}$ is regular if L is regular.

Exercise 22. The *perfect shuffle* of two strings x, $y \in \Sigma^*$ of equal lengths is the string of double length. If $x = x_1x_2 \cdots x_k$ and $y = y_1y_2 \cdots y_k$, the *perfect shuffle* $x \|_p y = x_1y_1x_2\, y_2 \cdots x_ky_k$. The *perfect shuffle* of two languages L_1 and L_2 is $L_1 \|_p L_2 = \{w \in \{a, b\}^* : w = x_1y_1x_2\, y_2 \cdots x_ky_k, \text{ where } x_1x_2 \cdots x_k \in L_1 \text{ and } y_1y_2 \cdots y_k \in L_2\}$. Show that the class of regular languages is closed under *perfect shuffle*.