Formal Language and Automata Theory (CS21004)

Tutorial - V

Class: CSE 2nd Year *Date*: 8th February, 2010

Exercise 1. Design a DFA that recognises the language $L = \{a^n : n \text{ is divisible by } 2 \text{ or } 5\}$.

Exercise 2. Give description of a DFA over $\Sigma = \{a, b\}$ that recognizes the language $L_3 = \{x \in \Sigma^* : |x|_a \geqslant 3 \text{ and } |x|_b \leq 2\}$.

Exercise 3. Let the alphabet be $\Sigma = \{a, b\}$.

a) Show that there is a regular language that cannot be recognised by a DFA with one final state.

b) Show that there is a regular language that cannot be recognised by a DFA with less than k final states.

Exercise 4. *Reverse* of a string x is denoted by x^R. It is a function $-^R : \Sigma^* \to \Sigma^*$ defined as follows:

$$x^R = \begin{cases} \varepsilon & \text{if } x = \varepsilon, \\ y^R\sigma & \text{if } x = \sigma y,\ \sigma \in \Sigma \text{ and } y \in \Sigma^*. \end{cases}$$

The *reverse* of a language L is denoted by L^R and is defined as $L^R = \{x^R \in \Sigma^* : x \in L\}$.
If L is regular, then so is L^R.

Exercise 5. Show that the language
$L = \{x \in \{0,1\}^* : x \text{ has equal number of } 01 \text{ and } 10 \text{ as substrings }\}$ is regular.

Exercise 6. Design NFA for the following languages and convert it to DFA.

$L_{5a} = \{x \in \{0,1\}^\star : \exists y \in \{0,1\}^\star \text{ such that } x = y1a1, \text{ where } a \in \{0,1\}\}$.

Exercise 7. Let Σ be an alphabet and $Reg_\Sigma = \{L \subseteq \Sigma^\star : L \text{ is regular}\}$. Prove the following results -

a) Reg_Σ is closed under Boolean operations - finite union, finite intersection, complementation with respect to $\Sigma^\star$.

b) Reg_Σ is closed under concatenation, Kleene-star, and reversal.

Exercise 8. Consider the language $L = \{x \in \{a, b\}^\star : |x|_a = |x|_b\}$ and prove that the following grammars are *sound* but not *complete*.

i. $S \to S01S \mid S10S \mid \varepsilon$,

ii. $S \to 0S1 \mid 1S0 \mid 01S \mid S01 \mid S10 \mid \varepsilon$.

Exercise 9. Give a CSG for the language $L = \{x = ww : w \in \{a, b\}^\star\}$.

Exercise 10. Give a CFG for the language $L = \{x \neq ww : w \in \{a, b\}^\star\}$. Note that the complement of L is not a CFL.