Formal Language and Automata Theory (CS21004)

Tutorial - I

Class: CSE 2nd Year (GB, DD, DP, MM) *Date*: 4th January, 2010

Note: The first tutorial contains a few problems of discrete mathematics as we have not yet started formal language theory.

Exercise 1. A *lucky number* is a positive integer which is 19 times the sum of its digits. How many different lucky numbers are there? [AMC I 2007 (28)]

Exercise 2. On my calculator screen the number 2659 can be read upside down as 6592. The digits that can be read upside down are 0, 1, 2, 5, 6, 8, 9 and are read as 0, 1, 2, 5, 9, 8, 6 respectively. Starting with 1, the fifth number that can be read upside down is 8 and the fifteenth is 21. What is the 2010^{th} number that can be read upside down? [similar to AMC I 2007 (30)]

Exercise 3. Let $h(n) = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \cdots\cdots + \frac{1}{n}$.
Prove that $n + h(1) + h(2) + \cdots\cdots + h(n-1) = \text{nh}(n)$, for all $n \geqslant 2$.

Exercise 4. Prove that for any positive integer n, there is a n-digit decimal number, containing only the digits 2 and 3, and divisible by 2^n.

Exercise 5. The n^{th} *Fermat number* F_n is $2^{2^n} + 1$, $n \geqslant 0$. Prove that $F_1 \cdot F_2 \cdot F_3 \cdot \cdots \cdot F_n = F_{n+1} - 2$. Can you conclude from this proposition that there are infinite number of primes?

Exercise 6. Find the sum of all 3-digit decimal numbers that contains at least one even digit and at least one odd digit. [RMO 2009 (3)]

Exercise 7. What is the smallest number of odd numbers in the range 1, ⋯⋯, 2008 such that, no matter how these numbers are chosen, there will always be two which add to 2010? [similar to AMC 2007 I (26)]

Exercise 8. All the vertexes of a 15-gon, not necessarily regular, lie on the circumference of a circle and the centre of this circle is inside the 15-gon. What is the largest possible number of obtuse-angled triangle where the vertexes of each triangle are vertexes of the 15-gon? [AMC 2008 I (30)]

Exercise 9. Let $\Sigma = \{a, b, c\}$, the collection of all finite length strings over Σ is called $\Sigma^\star = \{\varepsilon, a, b, c, aa, ab, ac, ba, bb, bc, ca, cb, cc, aaa, \cdots\cdots\}$. Show that there is a *one-to-one* and *onto* map from the set of natural numbers, $\mathbb{N}$, to $\Sigma^\star$. What is your conclusion about the size or cardinality of $\Sigma^\star$?

Exercise 10. A language L over a $\Sigma = \{a, b, c\}$ is a subset of $\Sigma^\star$ i.e. $L \subseteq \Sigma^\star$. A language L is finite if the number of strings of L is finite, otherwise it is infinite.

$L_1 = \{a^i : 0 \leqslant i \leqslant 100\} = \{\varepsilon, 1, aa, aaa, \cdots\cdots, \overbrace{aaa\cdots a}^{100}\}$ is finite and $L_2 = \{a^i : i \geqslant 0\} = \{\varepsilon, a, aa, aaa, \cdots\}$ is infinite. What can you conclude about the size of the collection of

i. all languages over $\Sigma^\star$,

ii. all finite languages (finite subset of $\Sigma^\star$) over $\Sigma^\star$?

AMC: Australian Mathematics Competition
RMO: Regional Mathematics Olympiad