# Representation of `int` data

## World Inside a Computer is Binary

## Decimal Number System

- Basic symbols: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9
- Radix-10 positional number system. The radix is also called the base of the number system.

$$12304 = 1\times10^4+2\times10^3+3\times10^2+0\times10^1+4\times10^0$$

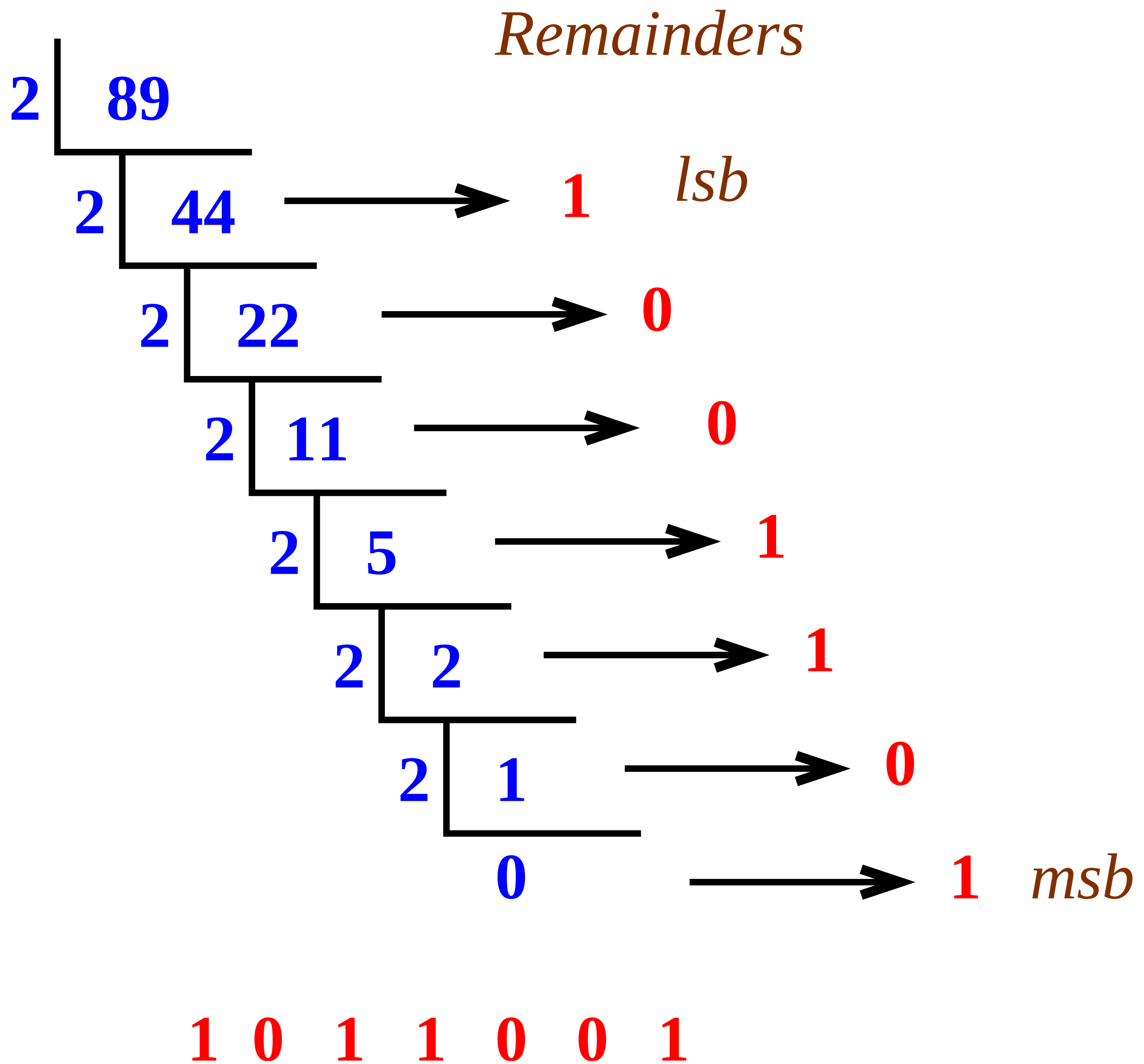
## Unsigned Binary Number System

- Basic symbols: 0, 1
- Radix-2 positional number system.

$$10110 = 1 \times 2^4 + 0 \times 2^3 + 1 \times 2^2 + 1 \times 2^1 + 0 \times 2^0$$

The value is 22 in decimal.

# Decimal to Binary Conversion


Remainders
2 89
2 44
1 lsb
2 22
0
2 11
0
2 5
1
2 2
1
2 1
0
0
1 msb
1 0 1 1 0 0 1

## Decimal to Binary Conversion

$$
\begin{aligned}
89_D &= 1\cdot 2^6 + 0\cdot 2^5 + 1\cdot 2^4 + 1\cdot 2^3 + 0\cdot 2^2 + 0\cdot 2^1 + 1\cdot 2^0 \\
&= 2\cdot(2\cdot(2\cdot(2\cdot(2\cdot(2\cdot 1 + 0) + 1) + 1) + 0) + 0) + 1
\end{aligned}
$$

# Finite `word` Size of the CPU

- 32-bit word:

| 31 | 30 | | 0 |
|---|---|---|---|
| | | $\cdots$ | |

- Largest Number: $\sum_{i=0}^{31} 2^i = 4294967295$,

| 31 | 30 | | 0 |
|---|---|---|---|
| 1 | 1 | $\cdots$ | 1 |

- Smallest Number: 0

| 31 | 30 | | 0 |
|---|---|---|---|
| 0 | 0 | $\cdots$ | 0 |

## Note

- The range of `unsigned int` or `unsigned` is (in 32-bits) `0` to `4294967295`.
- The range of `unsigned short int` or `unsigned short` is (in 16-bits) `0` to `65535`.

## An Example with 4-bit Word Size

| | | | |
|---|---|---|---|
| $b_3$ | $b_2$ | $b_1$ | $b_0$ |

The Range of unsigned integer is:

$$0 \text{ to } 2^4 - 1 = 15$$

| Bit String | | | | Decimal Value |
|---|---|---|---|---|
| $b_3$ | $b_2$ | $b_1$ | $b_0$ | |
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 1 | 1 |
| 0 | 0 | 1 | 0 | 2 |
| 0 | 0 | 1 | 1 | 3 |
| 0 | 1 | 0 | 0 | 4 |
| 0 | 1 | 0 | 1 | 5 |
| 0 | 1 | 1 | 0 | 6 |
| 0 | 1 | 1 | 1 | 7 |

| Bit String | | | | Decimal Value |
|---|---|---|---|---|
| $b_3$ | $b_2$ | $b_1$ | $b_0$ | |
| 1 | 0 | 0 | 0 | 8 |
| 1 | 0 | 0 | 1 | 9 |
| 1 | 0 | 1 | 0 | 10 |
| 1 | 0 | 1 | 1 | 11 |
| 1 | 1 | 0 | 0 | 12 |
| 1 | 1 | 0 | 1 | 13 |
| 1 | 1 | 1 | 0 | 14 |
| 1 | 1 | 1 | 1 | 15 |

## Signed Decimal Number

- We use '$+$' and '$-$' symbols to indicate sign of a decimal number.
- In a binary system only two symbols $\{0, 1\}$ are available to encode any information. So one extra bit is required to indicate the sign of a number.

## Three Popular Schemes

- Signed Magnitude,
- 1's Complement,
- 2's Complement

## Signed Magnitude

Consider a 4-bit word as an example.

| | | | |
|---|---|---|---|
| $b_3$ | $b_2$ | $b_1$ | $b_0$ |

$b_3$ is zero (0) for a positive number and it is one (1) for a negative number. Other three bits $b_2$ $b_1$ $b_0$ represent the magnitude of the number.

| Bit String | | | | Decimal Value |
|---|---|---|---|---|
| $b_3$ | $b_2$ | $b_1$ | $b_0$ | |
| 0 | 0 | 0 | 0 | +0 |
| 0 | 0 | 0 | 1 | +1 |
| 0 | 0 | 1 | 0 | +2 |
| 0 | 0 | 1 | 1 | +3 |
| 0 | 1 | 0 | 0 | +4 |
| 0 | 1 | 0 | 1 | +5 |
| 0 | 1 | 1 | 0 | +6 |
| 0 | 1 | 1 | 1 | +7 |

| Bit String | | | | Decimal Value |
|---|---|---|---|---|
| $b_3$ | $b_2$ | $b_1$ | $b_0$ | |
| 1 | 1 | 1 | 1 | -7 |
| 1 | 1 | 1 | 0 | -6 |
| 1 | 1 | 0 | 1 | -5 |
| 1 | 1 | 0 | 0 | -4 |
| 1 | 0 | 1 | 1 | -3 |
| 1 | 0 | 1 | 0 | -2 |
| 1 | 0 | 0 | 1 | -1 |
| 1 | 0 | 0 | 0 | -0 |

## Signed magnitude

- There are two representations of zero: $+0, -0$,
- The range is $[-7, \cdots, +7]$ in 4-bits.
- If the word size is $n$-bits, the range is $[-(2^{n-1} - 1), \cdots, +(2^{n-1} - 1)]$.

## 1's Complement Numeral

- Positive Numbers are same as the signed magnitude representation with the most significant bit[a] zero.
- If $n$ is a number in 1's complement form, $-n$ is obtained by changing every bit to its complement. The result is called the 1's complement of $n$.

[a] $b_3$ in case of 4-bits and $b_{n-1}$ in case of $n$-bits.

## 1's Complement Numeral

- A negative number has one (1) in its most significant bit.

$$\overbrace{0\ 1\ 1\ 0\ 1\ 1\ 0\ 1}^{+109} \xrightarrow{\text{1's complement}} \overbrace{1\ 0\ 0\ 1\ 0\ 0\ 1\ 0}^{-109}$$

$$\overbrace{1\ 0\ 0\ 1\ 0\ 0\ 1\ 0}^{-109} \xrightarrow{\text{1's complement}} \overbrace{0\ 1\ 1\ 0\ 1\ 1\ 0\ 1}^{+109}$$

| Decimal Value | Bit String $b_3$ | $b_2$ | $b_1$ | $b_0$ | Bit String $b_3$ | $b_2$ | $b_1$ | $b_0$ | Decimal Value |
|---|---|---|---|---|---|---|---|---|---|
| +7 | 0 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | −7 |
| +6 | 0 | 1 | 1 | 0 | 1 | 0 | 0 | 1 | −6 |
| +5 | 0 | 1 | 0 | 1 | 1 | 0 | 1 | 0 | −5 |
| +4 | 0 | 1 | 0 | 0 | 1 | 0 | 1 | 1 | −4 |
| +3 | 0 | 0 | 1 | 1 | 1 | 1 | 0 | 0 | −3 |
| +2 | 0 | 0 | 1 | 0 | 1 | 1 | 0 | 1 | −2 |
| +1 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 0 | −1 |
| +0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | −0 |

## 1's Complement Representation

- Two representations of zero: $+0, -0$.
- The range is $[-7, \cdots, +7]$ in 4-bits.
- The range is $[-(2^{n-1} - 1), \cdots, +(2^{n-1} - 1)]$.
- Positive number representation is identical to signed magnitude, but the negative number representations are different.

# Signed Magnitude Verses 1's Complement

| Decimal | Signed Magnitude | 1's Complement |
|---|---|---|
| −0 | 1000 | 1111 |
| −1 | 1001 | 1110 |
| −2 | 1010 | 1101 |
| −3 | 1011 | 1100 |
| −4 | 1100 | 1011 |
| −5 | 1101 | 1010 |
| −6 | 1110 | 1001 |
| −7 | 1111 | 1000 |

## 2's Complement

- Positive Numbers are same as the signed magnitude and 1's complement representations with the most significant bit[a] zero.
- If $n$ is a number in 2's complement form, $-n$ is obtained by changing every bit to its complement and finally adding one to the complemented number.

[a] $b_3$ in case of 4-bits and $b_{n-1}$ in case of $n$-bits.

## 2's Complement Numeral

A negative number has a one (1) in the most significant position.

$$\overbrace{0\ 1\ 1\ 0\ 1\ 1\ 0\ 1}^{+109} \xrightarrow{\text{1's complement}} 10010010 + 1$$

$$\xrightarrow{\text{2's complement}} \overbrace{1\ 0\ 0\ 1\ 0\ 0\ 1\ 1}^{-109}$$

$$\overbrace{1\ 0\ 0\ 1\ 0\ 0\ 1\ 1}^{-109} \xrightarrow{\text{1's complement}} 01101100 + 1$$

$$\xrightarrow{\text{2's complement}} \overbrace{0\ 1\ 1\ 0\ 1\ 1\ 0\ 1}^{+109}$$

## Direct 2's Complement

**1 0 1 1 0 1 0 1 1 0 0 0 0** ⟶ **0 1 0 0 1 0 1 0 1 0 0 0 0**

**2's Comp**

## 2's Complement Numeral

- Only one representations of zero: 0000.
- The range is $[-8, \cdots, +7]$ for 4-bits.
- For $n$-bits, the range is i$[-(2^{n-1}), \cdots, +(2^{n-1} - 1)]$.
- Positive representation is identical to signed magnitude and 1's complement, but the negative representation is different.

## `int, short int`

1. The range of data of type `int` (32-bits) is `-2147483648` to `2147483647`.
2. The range of `short int` (16-bits) is `-32768` to `32767`.
3. The range of `long long int` (64-bits) is `-9223372036854775808` to `9223372036854775807`.

| Decimal Value | Bit String $b_3$ | $b_2$ | $b_1$ | $b_0$ | Bit String $b_3$ | $b_2$ | $b_1$ | $b_0$ | Decimal Value |
|---|---|---|---|---|---|---|---|---|---|
| +7 | 0 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | −8 |
| +6 | 0 | 1 | 1 | 0 | 1 | 0 | 0 | 1 | −7 |
| +5 | 0 | 1 | 0 | 1 | 1 | 0 | 1 | 0 | −6 |
| +4 | 0 | 1 | 0 | 0 | 1 | 0 | 1 | 1 | −5 |
| +3 | 0 | 0 | 1 | 1 | 1 | 1 | 0 | 0 | −4 |
| +2 | 0 | 0 | 1 | 0 | 1 | 1 | 0 | 1 | −3 |
| +1 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 0 | −2 |
| 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | −1 |

# Signed Magnitude, 1's and 2's Complements

| Decimal | Sig. Mag. | 1's Compl. | 2's Compl. |
|---|---|---|---|
| $-0$ | 1000 | 1111 | |
| $-1$ | 1001 | 1110 | 1111 |
| $-2$ | 1010 | 1101 | 1110 |
| $-3$ | 1011 | 1100 | 1101 |
| $-4$ | 1100 | 1011 | 1100 |
| $-5$ | 1101 | 1010 | 1011 |
| $-6$ | 1110 | 1001 | 1010 |
| $-7$ | 1111 | 1000 | 1001 |
| $-8$ | | | 1000 |

## Interpretation of Bits

Consider a 8-bit 2's complement number. If the number is positive e.g. 01101010, the value is as usual

$2^6 + 2^5 + 2^3 + 2^1 = 64 + 32 + 8 + 2 = 106$ (decimal).

# Interpretation of Bits

If the number is negative e.g. 11101010, its value is − (10000 0000 − 1110 1010) i.e.

$$
\begin{aligned}
& -[2^8 - (2^7 + 2^6 + 2^5 + 2^3 + 2^1)] \\
= & -2^7 + 2^6 + 2^5 + 2^3 + 2^1 \\
= & -22(\text{in decimal}).
\end{aligned}
$$

**Weight of msb is −ve**

# Sign Bit Extension

| $-39$ | $25$ |
|---:|---:|
| 1011001 | 0011001 |
| 11011001 | 00011001 |
| 111011001 | 000011001 |
| 1111011001 | 0000011001 |
| 11111011001 | 00000011001 |
| 111111011001 | 000000011001 |

# 2's Complement Addition

| Binary | Decimal |
|---|---|
| 0011 | 3 |
| + 0010 | + 2 |
| 0101 | 5 |

| Binary | Decimal |
|---|---|
| 1101 | −3 |
| + 1110 | + −2 |
| ~~1~~ 1011 | −5 |

| Binary | Decimal |
|---|---|
| 0011 | 3 |
| + 1011 | + −5 |
| 1110 | −2 |

| Binary | Decimal |
|---|---|
| 0100 | 4 |
| + 0101 | + 5 |
| 1001 | −7 |

Overflow

## 2's Complement Addition

| | | | |
|---|---|---|---|
| | 1101 | | $-3$ |
| $+$ | 1010 | $+$ | $-6$ |
| 1 | 0111 | | 7 |

Overflow

There may be carry-out without overflow. There may not be overflow even if there is carry-out.

## `int` in Your Machine

The C data type `int` is a 32-bit 2's complement number. Its range is $-2147483648$ to $+2147483647$. If one (1) is added to the largest positive number, the result is the smallest negative number.

| | |
|---|---|
| 0111 1111 1111 1111 1111 1111 1111 1111<br>+1 | 2147483647 |
| 1000 0000 0000 0000 0000 0000 0000 0000 | $-2147483648$ |

## 10's Complement Number

The 2's complement numeral is nothing special. We can use radix-complement numerals for any radix to represent signed numbers without importing any new sign symbol. We consider radix-complement decimal or 10's complement numerals.

## 3-digit 10's Complement Numeral

There are one thousand patterns (000 to 999) with three decimal digits, we interpret them in the following way:

- If the most significant digit is any one of $\{0, 1, 2, 3, 4\}$, the denotation is a usual positive number e.g. 341 is same as usual decimal 341.

## 3-digit 10's Complement Numeral

- If the most significant digit is any one of $\{5, 6, 7, 8, 9\}$, the number is treated as a negative number ($n$).
- If $n$ is a 10's complement number, $-n$ is obtained by ordinary decimal subtraction $1000 - n$.

## 10's Complement Numeral

Consider the 3-digit 10's complement numeral 725. It is a negative number whose magnitude is $1000 - 725 = 275$. The range of numbers represented in 3 digits is $-10^3/2$ to $+10^3/2 - 1$. In $n$-digits the range is $-10^n/2$ to $+10^n/2 - 1$.

## Addition of 10's Complement Numeral

$$
\begin{array}{rr}
 & 127 \\
+ & 205 \\
\hline
 & 332 \\
 & 427 \\
+ & 305 \\
\hline
 & 732
\end{array}
\qquad
\begin{array}{rl}
 & 127 \\
+ & 821\ (-179) \\
\hline
 & 948\ (-52)
\end{array}
\qquad
\begin{array}{rrl}
 & & 821\ (-179) \\
+ & & 753\ (-247) \\
\hline
\not{1} & \leftarrow & 574\ (-426) \\
 & & 521\ (-179) \\
+ & & 753\ (-247) \\
\hline
\not{1} & \leftarrow & 274
\end{array}
$$

overflow

## Digit Extension

| 3-digit | 4-digit | 5-digit | Decimal value |
|---|---|---|---|
| 234 | 0234 | 00234 | 234 |
| 721 | 9721 | 99721 | $-279$ |