Computation of $sin(x)$

Power Series

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!} \cdots$$

Finite number of terms of this infinite series may be used to compute an approximate value of $\sin(x)$, where x is in radian.

$$\begin{aligned}
\sin x &= x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!} \cdots \\
&= \sum_{i \geq 0} (-1)^i \frac{x^{2i+1}}{(2i+1)!} \\
&= \sum_{i \geq 0} t_i, \text{ where } t_i = (-1)^i \frac{x^{2i+1}}{(2i+1)!}
\end{aligned}$$

Inductive Definition of t_i

$$t_i = \begin{cases} x & \text{if } i = 0, \\ -t_{i-1}\dfrac{x^2}{2i(2i+1)} & \text{if } i > 0. \end{cases}$$

This is also called recurrence relation or recursive definition of a term.

Note: Do not compute each term directly as

$$t_i = (-1)^i \frac{x^{2i+1}}{(2i+1)!}$$

Approximation of $\sin(x)$

The sum upto the n^{th} term (S_n) of the series gives an approximate value of $\sin(x)$. The inductive definition of S_n is

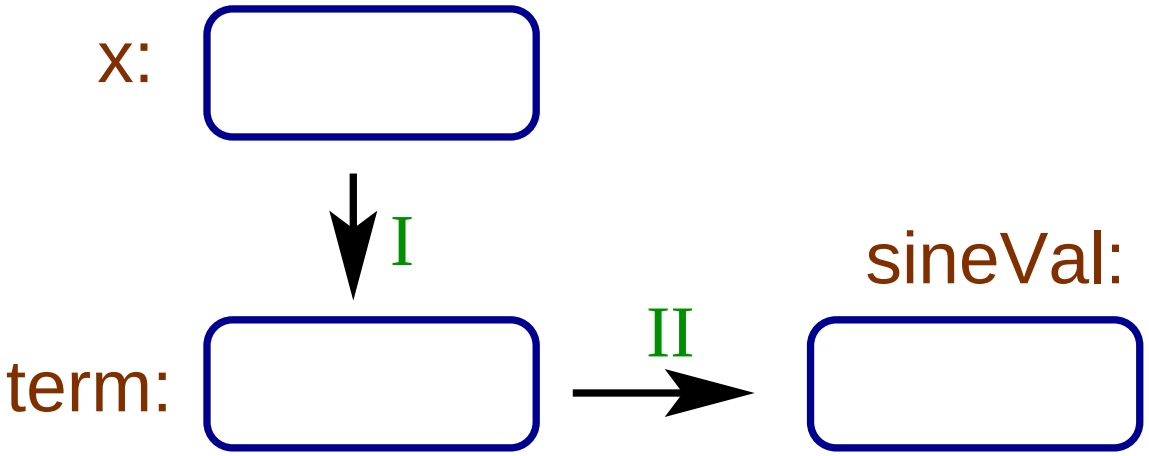
$$S_n = \begin{cases} t_0 & \text{if } n = 0, \\ s_{n-1} + t_n & \text{if } n > 0. \end{cases}$$

From Inductive Definition to Iterative Process

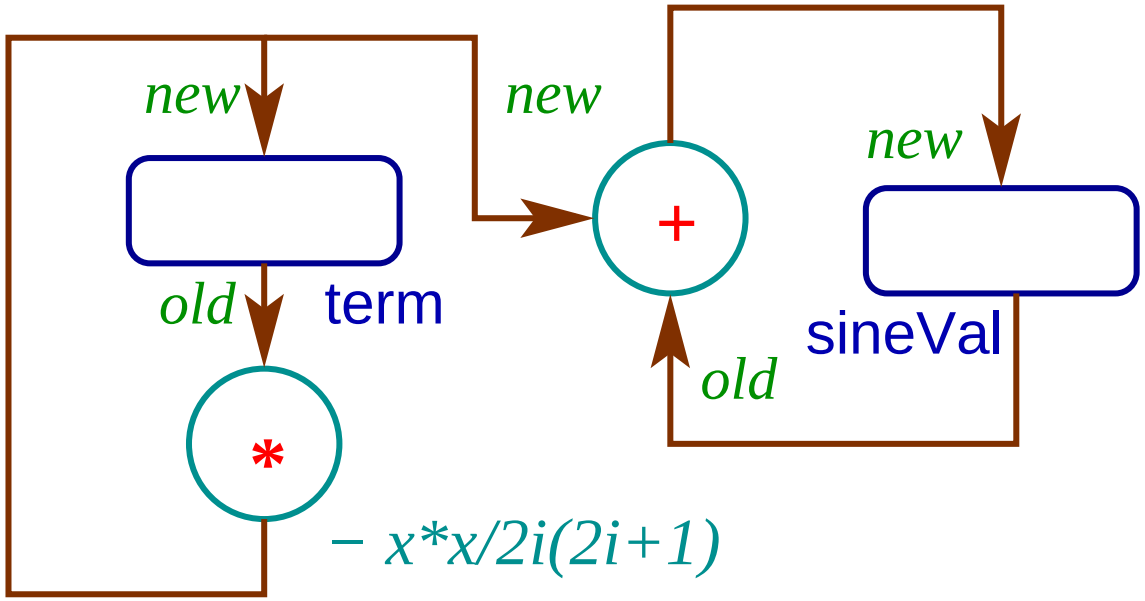
An iterative process of computation can be obtained from the inductive definition.

1. Start from initial values of t_i and S_i.
2. Repeat the following two steps.
   (a) Compute the next term t_{i+1}.
   (b) Compute the next approximate value of $\sin(x)$ by computing S_{i+1}.

Initialization



Iteration



Termination of Iteration

The process is to be terminated after a finite number of iterations. The termination may be

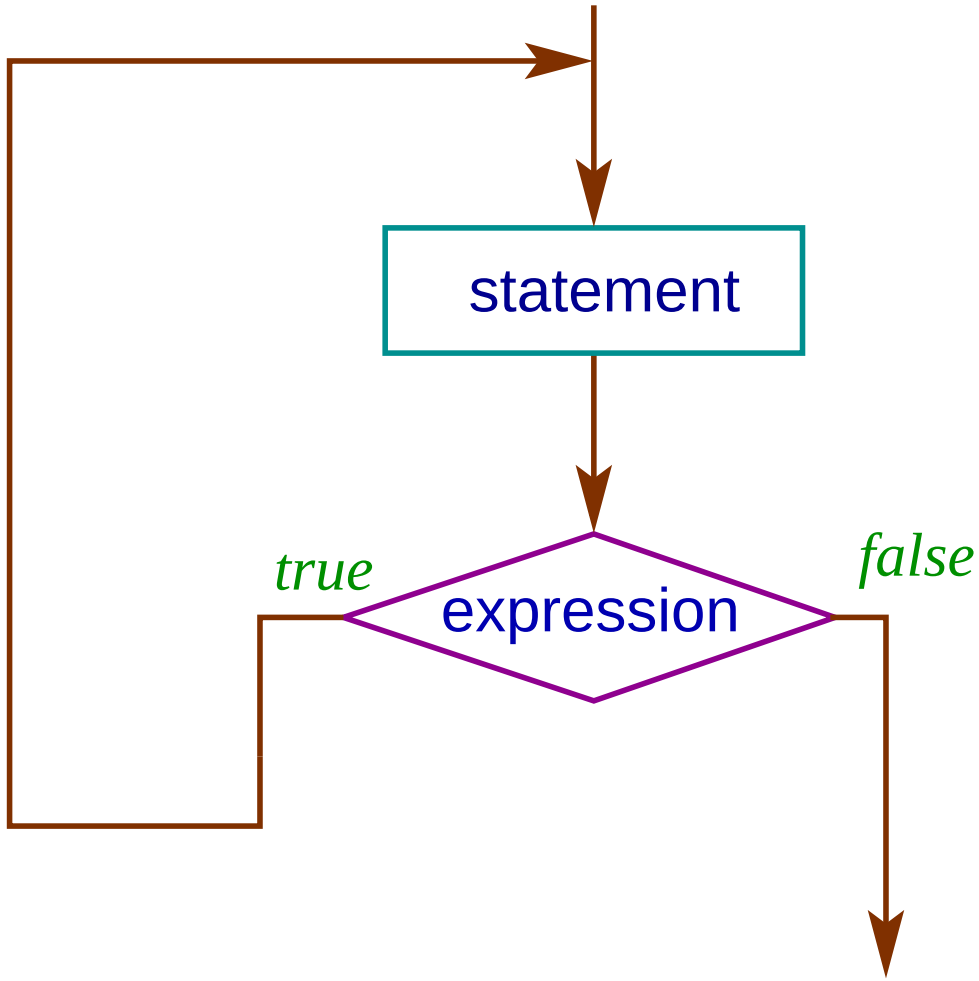
1. after a fixed number of iterations, or
2. after achieving a pre-specified accuracy.

do-while Statement

We shall use another iterative construct of C language to solve this problem.

```
do statement while (expression) ;
```

do-while Loop



```
/*
 * sin(x) using do-while
 * cc -Wall sin.c -lm
 */
#include <stdio.h>
#include <math.h>
int main()
{
 float x, precError, compError, xRadian, term, sineVal
 int termNo = 1 ;

 printf("Enter the value of an angle in Degree: ") ;
 scanf("%f", &x) ;
 printf("\nEnter the Percentage Error: ") ;
```

```
scanf("%f", &precError) ;

xRadian = M_PI*x/180.0 ;
term = xRadian ; // Initialization
sineVal = term ; //

do {                // Iteration
    float factor ;

    factor = 2.0 * termNo++ ;
    factor *= (factor + 1.0) ;
    factor = - xRadian * xRadian / factor ;
    sineVal += (term = factor * term) ;
    compError = 100.0*fabs(term/sineVal) ;
```

```
  } while (compError >= precError) ;

  printf("\nsin(%f) = %f\nNo. of Iterations = %d\n",
                              x, sineVal, termNo - 1) ;
  return 0 ;
} // sin.c
```

Statement in do-while

```
{
      float factor ;

      factor = 2.0 * termNo++ ;
      factor *= (factor + 1.0) ;
      factor = - xRadian * xRadian / factor ;
      sineVal += (term = factor * term) ;
      compError = 100.0*fabs(term/sineVal) ;
}
```

Expression in `do-while`

```
compError >= precError
```

Error

The program does not work for large value of angle due to error propagation (?).

```
$ ./a.out
Enter the value of an angle in Degree: 60
Enter the Percentage Error: 0.001
sin(60.000000) = 0.866026
No. of Iterations = 4
$
```

Error

```
$ ./a.out
Enter the value of an angle in Degree: 720
Enter the Percentage Error: 0.001
sin(720.000000) = 0.002326
No. of Iterations = 23
$ ./a.out
Enter the value of an angle in Degree: 1440
Enter the Percentage Error: 0.001
sin(1440.000000) = -225.040436
No. of Iterations = 36
$
```