# Merge Sort and Analysis
--- Analyzing Recursive Programs

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# Why are we dealing with merge sort in this course?

- It is a powerful application of Divide and Conquer technique in problem solving
- Also radically different from insertion, selection and bubble sort.
- We can now apply the techniques and compare the performance of this sort with the previous ones

# Merging

- Produce a sorted list from two *sorted* lists
- A simple way is to examine from the front.
- At each step find the smaller of the two elements at the current fronts
- Choose that element as the next element of the merged list.
- Remove the chosen element from its list, exposing the next element as the now first element.

# Example of Merging Iteratively

| L1 | L2 | M |
|---|---|---|
| 1,2,7,7,9 | 2,4,7,8 | Empty |
| 2,7,7,9 | 2,4,7,8 | 1 |
| 7,7,9 | 2,4,7,8 | 1,2 |
| 7,7,9 | 4,7,8 | 1,2,2 |
| 7,7,9 | 7,8 | 1,2,2,4 |
| 7,9 | 7,8 | 1,2,2,4,7 |
| 9 | 7,8 | 1,2,2,4,7,7 |
| 9 | 8 | 1,2,2,4,7,7,7 |
| 9 | Empty | 1,2,2,4,7,7,7,8 |
| Empty | Empty | 1,2,2,4,7,7,7,8,9 |

# Implementation of Merge Sort (MS)

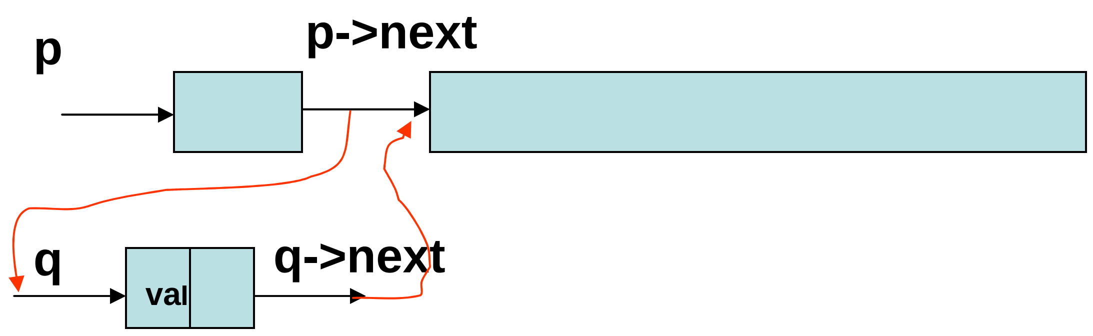
- It is easier to conceive in terms of a Linked List (LL)
- We have seen LLs in the lab.
- It is represented by a **node** (cell), which has two components: one *info* and the other *pointer* to the next element.

***<u>Definition of the nodes in C</u>***

```
struct node{
      int info;
      struct node *next;
 };
typedef struct node *NODEPTR;
```

# Inserting elements into the LL

```
#include<malloc.h>
NODEPTR getnode()
{
 NODEPTR p;
 p=(NODEPTR)malloc(sizeof(NODEPTR));
 return(p);
}

void insert(NODEPTR p,int val)
{
 NODEPTR q;
 q=getnode();
 q->info=val;
 q->next=p->next;
 p->next=q;
}
```



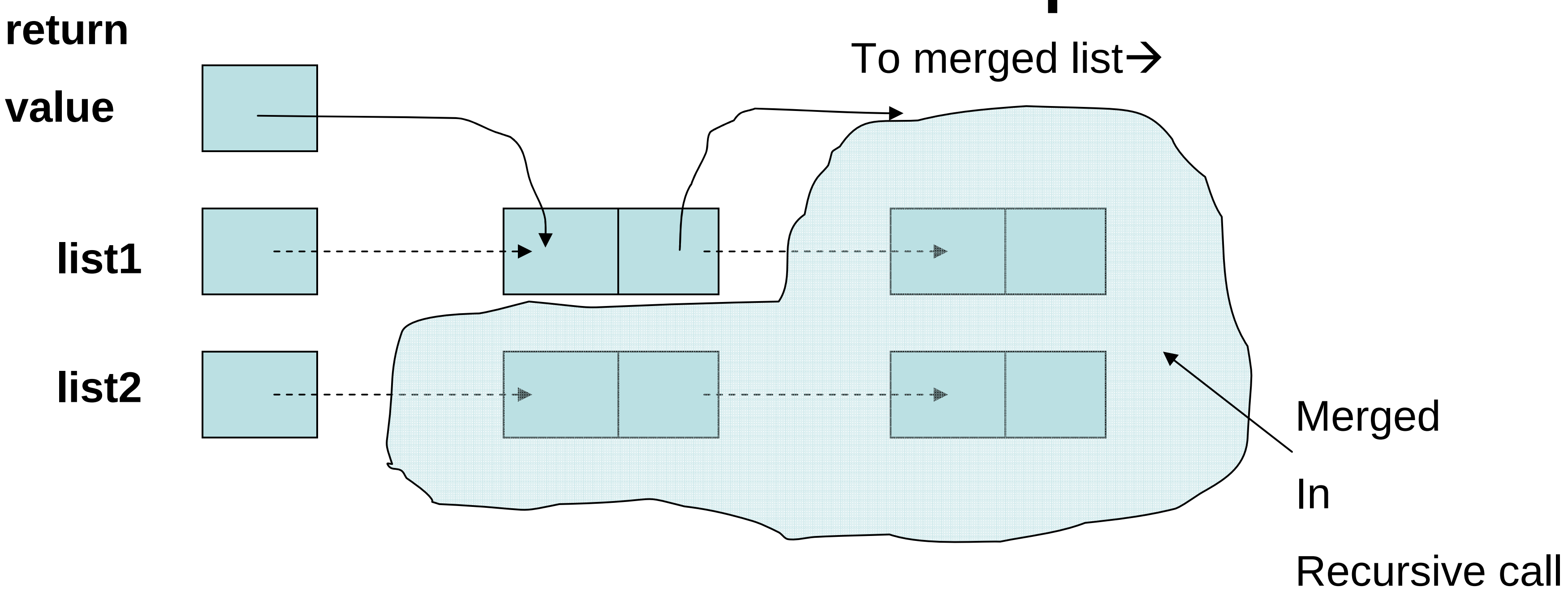
# Traversing the LL

```
void print(NODEPTR p)
{
  NODEPTR q;
  for(q=p->next;q!=NULL;q=q->next)
  {
    printf("%d \t",q->info);
  }
}
```

# Merging

```
NODEPTR merge(NODEPTR list1, NODEPTR list2)
{
 if(list1==NULL) return(list2);
 else if(list2==NULL) return(list1);

 else if(list1->info<=list2->info){
  list1->next=merge(list1->next,list2);
  return(list1);
 }
 else{
    list2->next=merge(list1,list2->next);
    return(list2);
 }
}
```

# Pictorial Description




- Dotted line represents initial list. After the recursive calls merge create the solid lines.

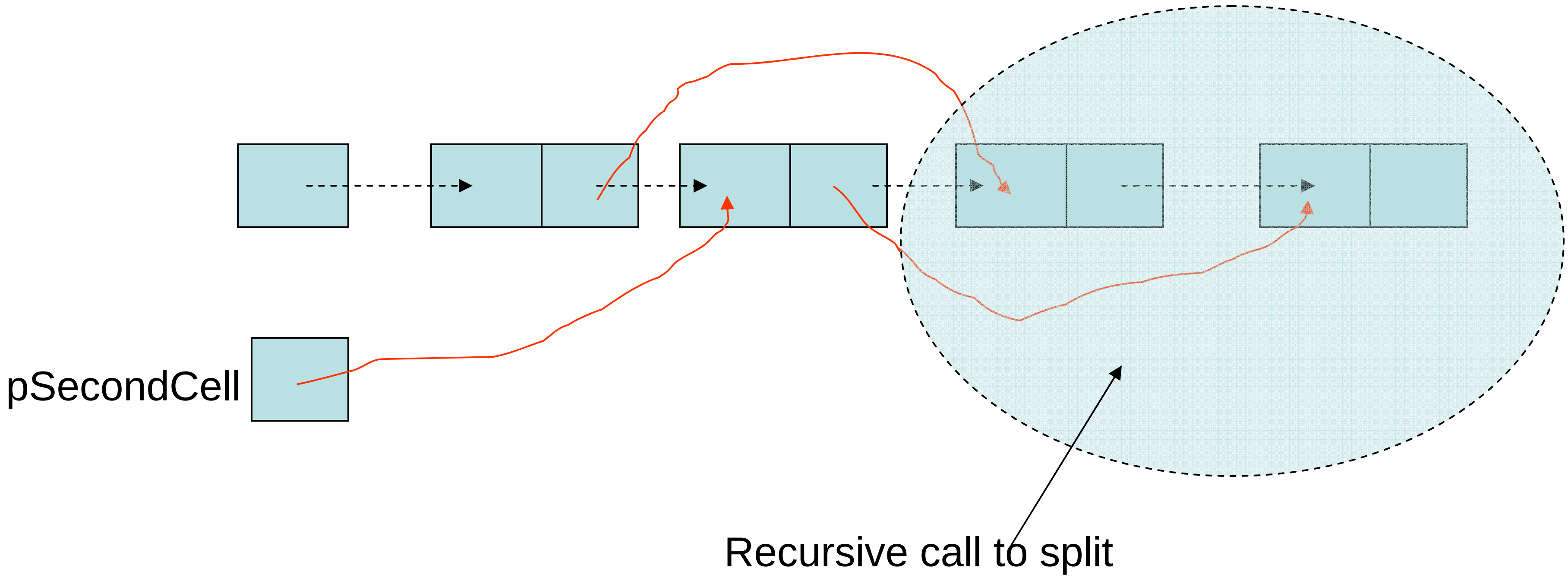
# Recursive calls to merge

| Call | Return |
|---|---|
| merge(12779,2478) | 122477789 |
| merge(2779,2478) | 22477789 |
| merge(779,2478) | 2477789 |
| merge(779,478) | 477789 |
| merge(779,78) | 77789 |
| merge(79,78) | 7789 |
| merge(9,78) | 789 |
| merge(9,8) | 89 |
| merge(9,NULL) | 9 |

# Splitting the list into 2 equal parts

```
NODEPTR split(NODEPTR list)
{
 NODEPTR pSecondCell;
 if(list==NULL) return(NULL);
 else if(list->next==NULL) return(NULL);
 else{//list contains more than two elements
     pSecondCell=list->next;
     list->next=pSecondCell->next;
     pSecondCell->next=split(pSecondCell->next);
     return(pSecondCell);
    }
}
```

# Pictorial representation of split



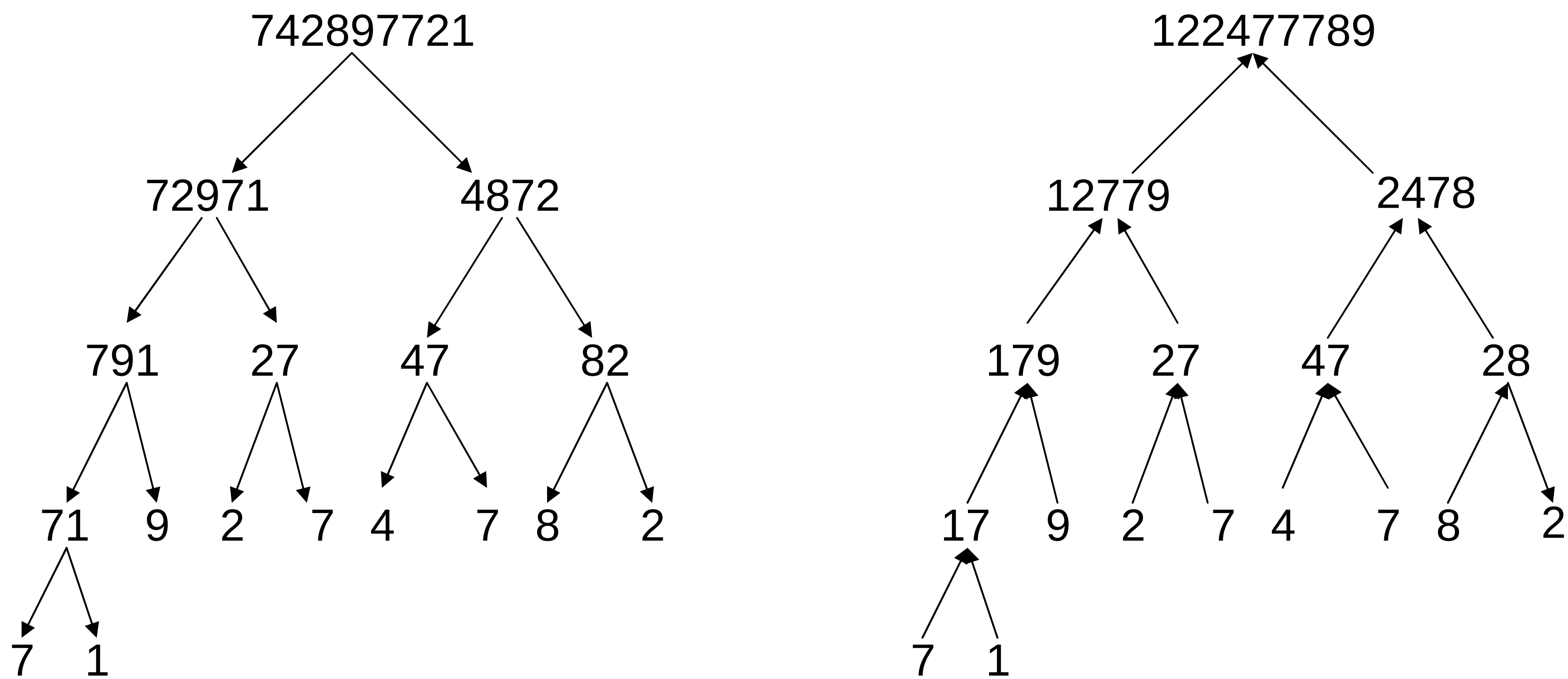
# The sorting algorithm

```
NODEPTR MergeSort(NODEPTR list)
{
 NODEPTR SecondList;
 NODEPTR printlist;
 static int i=0;
 i=i+1;
  printlist=getnode();
  printlist->next=list;
  printf("\n Rec No %d: The curr list is \n",i);
  print(printlist);

 if(list==NULL) return(NULL);
 else if(list->next==NULL) return(list);
 else{//at least two elements on list
  SecondList=split(list);

  return(merge(MergeSort(list),MergeSort(SecondList)));
 }
}
```

# Snap of the main

```
NODEPTR list;
 list=getnode();
 list->next=NULL;
while(val!=-1)
  {
     scanf("%d",&val);
     insert(list,val);
  }
 printf("The entered list is:\n");
 print(list);

 list->next=MergeSort(list->next);
 printf("\nThe sorted list is:\n");
 print(list);
 }
```

# Splitting and Merging Recursively



# Analyze the merge function

```
NODEPTR merge(NODEPTR list1, NODEPTR list2)
{
 if(list1==NULL) return(list2);                    O(1)+O(1)
 else if(list2==NULL) return(list1);               O(1)+O(1)

 else if(list1->info<=list2->info){                O(1)
  list1->next=merge(list1->next,list2);            O(1)+T(n-1)
  return(list1);                                   O(1)
 }
 else{
   list2->next=merge(list1,list2->next);           O(1)+T(n-1)
   return(list2);                                  O(1)
 }
}
```

*n is the input size of the problem; n is the sum of the two lists which are merged*

**$T(1)=O(1), T(n)=O(1)+T(n-1)$**

# RR for merge

- T(n)=T(n-1)+O(1) => T(n)=O(n)
- This is a 1-LiNoReCoCo
- CE has roots, r=1with multiplicity 1.
- F(n)=O(1). Thus the particular solution is P(n)=n(C)
- Thus T(n)=$C_1$+nC=O(n)

# Analyze the split

```
NODEPTR split(NODEPTR list)
{
 NODEPTR pSecondCell;
 if(list==NULL) return(NULL);                          O(1)
 else if(list->next==NULL) return(NULL);               O(1)+O(1)
 else{//list contains more than two elements
     pSecondCell=list->next;                            O(1)
     list->next=pSecondCell->next;                      O(1)
     pSecondCell->next=split(pSecondCell->next); O(1)+T(n-2)
     return(pSecondCell);                               O(1)
    }
}
```

*n is the length of the list which is being split*

**$T(0)=T(1)=O(1)$; $T(n)=T(n-2)+O(1) \Rightarrow T(n)=O(n)$**

# Analyzing merge sort

```
NODEPTR MergeSort(NODEPTR list)
{

 if(list==NULL) return(NULL);                        O(1)
 else if(list->next==NULL) return(list);             O(1)
 else{//at least two elements on list
  SecondList=split(list);                            O(1)+O(n)

  return(merge(MergeSort(list),MergeSort(SecondList)));
                                                      2T(n/2)+O(n)
 }
}
```

n is the number of elements in the list

**T(1)=O(1);T(n)=2T(n/2)+O(n)**

# Solve the RR

- $T(n)=2T(n/2)+O(n)$
- By Master's Theorem, a=2, b=2, d=1
- We have $\log_2 2=1$
- So, we have *$T(n)=O(n\log n)$*
- *Thus we have a better sorting algorithm in the worst case than the previous sorting algorithms we have seen.*
- *In fact we have T(n)=* $\Theta$**(nlogn)**

# Further Discussions

- We have also discussed on a Divide and Conquer strategy to solve the closest pair problem of n Cartesian points, better than the straight forward $O(n^2)$ algorithm
- Recurrence relation was:
  - $T(n)=2T(n/2)+7n$ which leads to $T(n)=O(nlogn)$