CS210 : FOUNDATIONS OF COMPUTER SCIENCE
Dept of Computer Sc and Engg, IIT Madras
Answer All Questions   Time: 2 hours
Full Marks 40

1. Prove the equivalence of the propositions defined in (a) and (b):
    (a) $(p \Rightarrow q) \vee (\neg p \Rightarrow q)$
    (b) $(\neg p \Leftrightarrow \neg q) \Leftrightarrow (p \Leftrightarrow q)$
What are such kinds of propositions called?

<div align="right">(3+1=4 marks)</div>

2. Consider the following 99 statements:
    S1: At least 1 of the statements are false
    S2: At least 2 of the statements are false
    …
    S99: At least 99 of the statements are false

What conclusion can you draw from the above statements. Explain your answer briefly.

<div align="right">(4 marks)</div>

3. (a) Write the following statement so that the negation appears only within the predicates, i,e. no negation is outside a quantifier: $\neg \forall x \forall y P(x, y)$
    (b) Prove or disprove the following equivalence:
        $\forall x P(x) \wedge \forall x Q(x) \Leftrightarrow \forall x \exists y (P(x) \vee Q(y))$

<div align="right">(2 x 2 = 4 marks)</div>

4. Prove that it is impossible to cover completely with dominos (1×2 pieces) the 8×8 chess-board with two squares at the opposite corners of the board removed. Hint: Adjacent squares of a chess-board have two colors, white and black.

<div align="right">(4 marks)</div>

5. Suppose that five ones and four zeros are arranged around a circle. Between any two equal bits you insert a 0 and between any two unequal bits you insert 1 and form nine new bits. Then erase the nine original bits. Show that when you iterate in this fashion, you can never reach nine zeros.

<div align="right">(4 marks)</div>

6. Determine the number of ways to cover a 2 by n board with dominos (1×2 pieces). What is the order of your answer?

<div align="right">(4 marks)</div>

7. (a) Suppose that R_1 and R_2 are equivalence relations on set S. State with brief reasons which of the following are also equivalence relations:
    (a) $R_1 \cap R_2$

(b) $R_1 \cup R_2$

(2 marks)

(b) Prove or disprove that the relation R on the set of integers defined as:
R={(a,b)|a>b} is an equivalence relation.

(1 mark)

(c) Comment on: "The Data Acyclic Graph of an equivalence relation is complete".

(1 mark)

8. Let A={1,2,3,4} and let R and S be the relations on set A:

R={(1,4),(3,2),(4,3)} and S={(1,1),(1,2),(2,2),(3,3),(4,2),(4,4)}.

Using Warshall's algorithm compute the transitive closure of $R \cup S$.

(4 marks)

9. Consider the following program:
SUBROUTINE COMP(X,Y;Z)
1. Z=X
2. W=Y
3. while(W>0)
    a. Z=Z+Y
    b. W=W-1
4. return
END OF SUBROUTINE

Show by mathematical induction that the given algorithm, correctly used, maintains the following loop invariant and produces the following output.

LOOP INVARIANT: $(Y \times W) + Z = X + Y^2$
OUTPUT : $Z = X + Y^2$

(4 marks)

10. The Halting problem discussed in the class states: "*Given a description of a program and a finite input, decide whether the program finishes running or will run forever, given that input*". The problem was shown to be unsolvable. However we have also seen in the class that we can prove that a specific instance of a program with specific inputs terminate. Is there is an apparent contradiction between the two? Explain in not more than two statements.

(4 marks)