CS210 : FOUNDATIONS OF COMPUTER SCIENCE
Dept of Computer Sc and Engg, IIT Madras
Answer All Questions Time: 3 hours
Full Marks 60

1. Determine the number of Permutations of {1,2,…,8} in which no even integer is in its normal position.

(5 marks)

2. Determine the number of integral solutions of the equation

$$x_1 + x_2 + x_3 + x_4 = 20$$

which satisfy $1 \le x_1 \le 6, 0 \le x_2 \le 7, 4 \le x_3 \le 8, 2 \le x_4 \le 6$.

(5 marks)

3. The line segments joining 10 points are arbitrarily colored red or blue. Prove that there must exist 3 points such the 3 line segments joining them are all red, or 4 points such that the 6 line segments joining them are all blue.

(5 marks)

4. Use the Pigeon Hole Principle to prove that the decimal expansion of a rational number m/n eventually has a repeating sequence.
For example: 34,478/99,000=0.34512512512512….

(5 marks)

5. Prove that the digraph of a partial order has no cycle of length greater than 1.

(5 marks)

6. Given a 0-1 matrix representation of a relation, denoted by M_R, how shall you determine whether the relation is transitive?

(5 marks)

7. Construct the Hasse Diagram for the relation on A={1,2,3,4,5} whose matrix is shown below:

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Does it form a lattice? What is the least and greatest element of the Poset A? Can you state a technique to determine from the matrix representation of a relation the least and greatest element of the Poset if they exist? Compute the least and greatest element of the above Poset using your stated method and cross-check your answer.

(1+1+1+2=5marks)

8. Consider the following conditional statement:
p: If the flood destroys my house or the fire destroys my house, then my insurance company will pay me.

Now consider the following statements:

S1: If my insurance company pays me, then the flood destroys my house or the fire destroys my house.

S2: If my insurance company pays me, then the flood destroys my house and the fire destroys my house.

S3 If my insurance company does not pay me, then the flood does not destroy my house or the fire does not destroy my house.

S4: If my insurance company does not pay me, then the flood does not destroy my house and the fire does not destroy my house.

Which of the statements, S1, S2 , S3 and S4 is the converse and contrapositive of the statement p? State the inverse statement of p.

(1.5+2+1.5=5marks)

9. Use finite sets and set operations to characterize the following languages over $\sum = \{a,b\}$. *For example, the set of strings of even length is {aa,ab,ba,bb}* and the set of strings of odd length is {aa,ab,ba,bb}*{a,b} (Recall Kleene closure notation described in class).*

(a) The set of strings which either begin with an ‘a’ or end with the substring ‘bbab’ or both.

(b) The set of strings which contains exactly one ‘a’ and at least three consecutive occurrences of ‘b’.

(2.5+2.5=5marks)

10. Consider linked lists which end in a loop (i.e., if you were to traverse such a linked list of n nodes then your traversal will never terminate i.e., there is no node whose NEXT link points to NULL. Like figure "6"). The tuple (n,c), where n denotes the total number of nodes in the linked list and c denotes the total number of nodes in the cycle is called the *configuration* of the list. We have to design an algorithm to compute the configuration (n,c) of the linked list, using only constant additional memory. Answer the following:

(a) Prove that the total number of possible configurations is countable.

(b) Describe an algorithm which takes as input the pointer LIST to the first element of the linked list and verifies if the configuration of the linked list is (n=N,c=C) for some positive integers N,C using constant additional memory.

(c) Design an algorithm which takes as input the pointer LIST and determines the configuration of the linked list using constant additional memory. What is the running time of your algorithm?

(3+3+4=10 marks)

11. The following procedure can be used to select the larger n of 2n distinct numbers:

1. Arrange n of the 2n numbers in ascending order. Denote the numbers $a_1, a_2, \ldots, a_n$ such that $a_1 < a_2 < \ldots < a_n$

2. Arrange the remaining n numbers in descending order. Denote the numbers $b_1, b_2, \ldots, b_n$ such that $b_1 > b_2 > \ldots > b_n$

3. Compare a_i and b_i and select the larger one of the two for i=1,2,…n.

Prove that the procedure is correct.

(5 marks)