Merge Sort and Analysis

--- Analyzing Recursive Programs

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Why are we dealing with merge sort in this course?

- It is a powerful application of Divide and Conquer technique in problem solving
- Also radically different from insertion, selection and bubble sort.
- We can now apply the techniques and compare the performance of this sort with the previous ones

Merging

- Produce a sorted list from two *sorted* lists
- A simple way is to examine from the front.
- At each step find the smaller of the two elements at the current fronts
- Choose that element as the next element of the merged list.
- Remove the chosen element from its list, exposing the next element as the now first element.

Example of Merging Iteratively

L1	L2	M
1,2,7,7,9	2,4,7,8	Empty
2,7,7,9	2,4,7,8	1
7,7,9	2,4,7,8	1,2
7,7,9	4,7,8	1,2,2
7,7,9	7,8	1,2,2,4
7,9	7,8	1,2,2,4,7
9	7,8	1,2,2,4,7,7
9	8	1,2,2,4,7,7,7
9	Empty	1,2,2,4,7,7,7,8
Empty	Empty	1,2,2,4,7,7,7,8,9

Implementation of Merge Sort (MS)

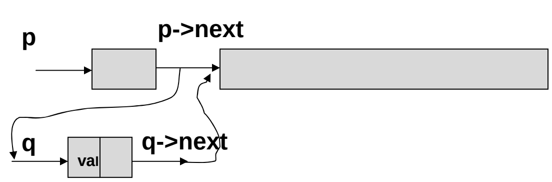
- It is easier to conceive in terms of a Linked List (LL)
- We have seen LLs in the lab.
- It is represented by a **node** (cell), which has two components: one *info* and the other *pointer* to the next element.

Definition of the nodes in C

```
struct node{
     int info;
     struct node *next;
 };
typedef struct node *NODEPTR;
```

Inserting elements into the LL

```
#include<malloc.h>
NODEPTR getnode()
{
 NODEPTR p;
 p=(NODEPTR)malloc(sizeof(NODEPTR));
 return(p);
}

void insert(NODEPTR p,int val)
{
 NODEPTR q;
 q=getnode();
 q->info=val;
 q->next=p->next;
 p->next=q;
}
```



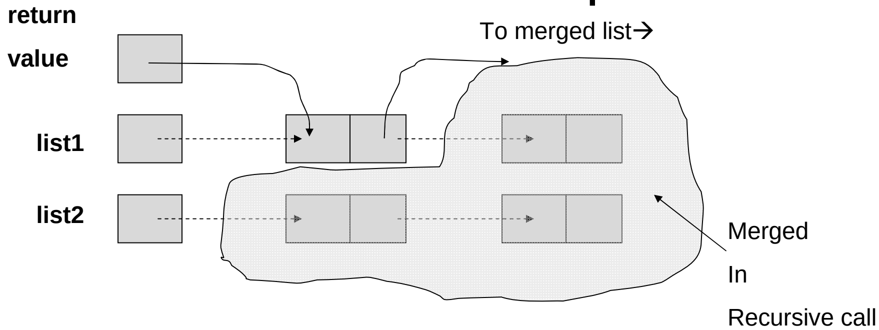
Traversing the LL

```
void print(NODEPTR p)
{
  NODEPTR q;
  for(q=p->next;q!=NULL;q=q->next)
  {
    printf("%d \t",q->info);
  }
}
```

Merging

```
NODEPTR merge(NODEPTR list1, NODEPTR list2)
{
 if(list1==NULL) return(list2);
 else if(list2==NULL) return(list1);

 else if(list1->info<=list2->info){
  list1->next=merge(list1->next,list2);
  return(list1);
 }
 else{
    list2->next=merge(list1,list2->next);
    return(list2);
 }
}
```

Pictorial Description



- Dotted line represents initial list. After the recursive calls merge create the solid lines.

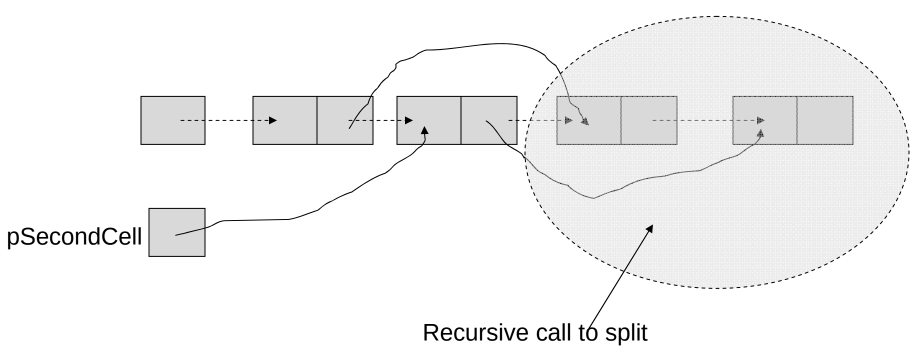
Recursive calls to merge

Call	Return
merge(12779,2478)	122477789
merge(2779,2478)	22477789
merge(779,2478)	2477789
merge(779,478)	477789
merge(779,78)	77789
merge(79,78)	7789
merge(9,78)	789
merge(9,8)	89
merge(9,NULL)	9

Splitting the list into 2 equal parts

```
NODEPTR split(NODEPTR list)
{
 NODEPTR pSecondCell;
 if(list==NULL) return(NULL);
 else if(list->next==NULL) return(NULL);
 else{//list contains more than two elements
     pSecondCell=list->next;
     list->next=pSecondCell->next;
     pSecondCell->next=split(pSecondCell->next);
     return(pSecondCell);
    }
}
```

Pictorial representation of split



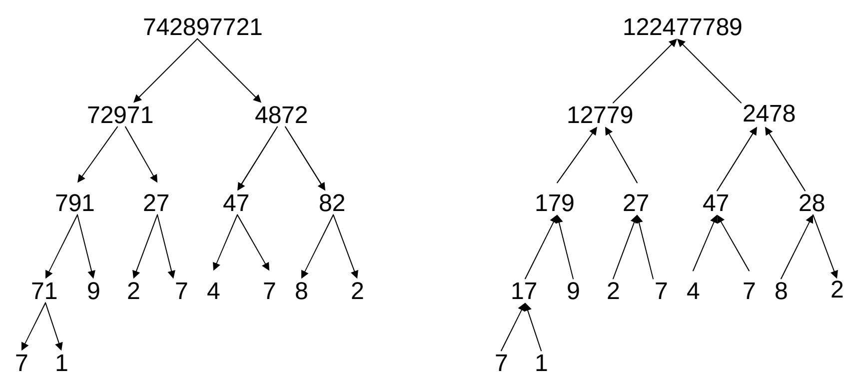
The sorting algorithm

```
NODEPTR MergeSort(NODEPTR list)
{
 NODEPTR SecondList;
 NODEPTR printlist;
 static int i=0;
 i=i+1;
  printlist=getnode();
  printlist->next=list;
  printf("\n Rec No %d: The curr list is \n",i);
  print(printlist);

 if(list==NULL) return(NULL);
 else if(list->next==NULL) return(list);
 else{//at least two elements on list
  SecondList=split(list);

  return(merge(MergeSort(list),MergeSort(SecondList)));
 }
}
```

Snap of the main

```
NODEPTR list;
 list=getnode();
 list->next=NULL;
while(val!=-1)
  {
     scanf("%d",&val);
     insert(list,val);
  }
 printf("The entered list is:\n");
 print(list);

 list->next=MergeSort(list->next);
 printf("\nThe sorted list is:\n");
 print(list);
 }
```

Splitting and Merging Recursively



Analyze the merge function

```
NODEPTR merge(NODEPTR list1, NODEPTR list2)
{
 if(list1==NULL) return(list2);                    O(1)+O(1)
 else if(list2==NULL) return(list1);               O(1)+O(1)

 else if(list1->info<=list2->info){                O(1)
  list1->next=merge(list1->next,list2);            O(1)+T(n-1)
  return(list1);                                   O(1)
 }
 else{
    list2->next=merge(list1,list2->next);          O(1)+T(n-1)
    return(list2);                                 O(1)
 }
}
```

n is the input size of the problem; n is the sum of the two lists which are merged

T(1)=O(1),T(n)=O(1)+T(n-1)

RR for merge

- T(n)=T(n-1)+O(1) => T(n)=O(n)
- This is a 1-LiNoReCoCo
- CE has roots, r=1with multiplicity 1.
- F(n)=O(1). Thus the particular solution is P(n)=n(C)
- Thus $T(n)=C_1+nC=O(n)$

Analyze the split

```
NODEPTR split(NODEPTR list)
{
 NODEPTR pSecondCell;
 if(list==NULL) return(NULL);                    O(1)
 else if(list->next==NULL) return(NULL);         O(1)+O(1)
 else{//list contains more than two elements
     pSecondCell=list->next;                     O(1)
     list->next=pSecondCell->next;               O(1)
     pSecondCell->next=split(pSecondCell->next); O(1)+T(n-2)
     return(pSecondCell);                        O(1)
    }
}
```

n is the length of the list which is being split

T(0)=T(1)=O(1); T(n)=T(n-2)+O(1)=> T(n)=O(n)

Analyzing merge sort

```
NODEPTR MergeSort(NODEPTR list)
{

 if(list==NULL) return(NULL);                          O(1)
 else if(list->next==NULL) return(list);               O(1)
 else{//at least two elements on list
  SecondList=split(list);                              O(1)+O(n)

  return(merge(MergeSort(list),MergeSort(SecondList)));
                                                        2T(n/2)+O(n)
 }
}
```

n is the number of elements in the list
T(1)=O(1);T(n)=2T(n/2)+O(n)

Solve the RR

- $T(n)=2T(n/2)+O(n)$
- By Master's Theorem, a=2, b=2, d=1
- We have $\log_2 2=1$
- So, we have *T(n)=O(nlogn)*
- *Thus we have a better sorting algorithm in the worst case than the previous sorting algorithms we have seen.*
- *In fact we have T(n)=* **Θ(nlogn)**