Symmetric Key Ciphers

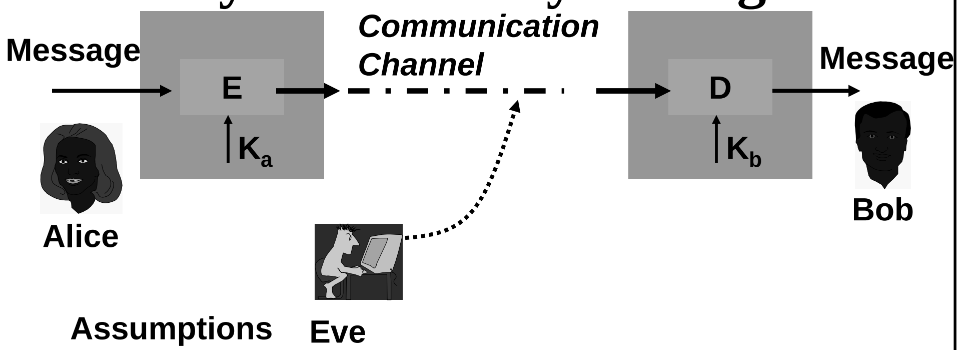
Debdeep Mukhopadhyay

Assistant Professor
Department of Computer Science and Engineering
Indian Institute of Technology Kharagpur
INDIA -721302

Objectives

- Definition of Symmetric Types of Symmetric Key ciphers
  - Modern Block Ciphers
- Full Size and Partial Size Key Ciphers
- Components of a Modern Block Cipher
  - PBox (Permutation Box)
  - SBox (Substitution Box)
  - Swap
  - Properties of the Exclusive OR operation
- Diffusion and Confusion
- Types of Block Ciphers: Feistel and non-Feistel ciphers

Symmetric Key Setting



Assumptions

K_a is the encryption key, K_b is the decryption key.

For symmetric key ciphers, $K_a=K_b$

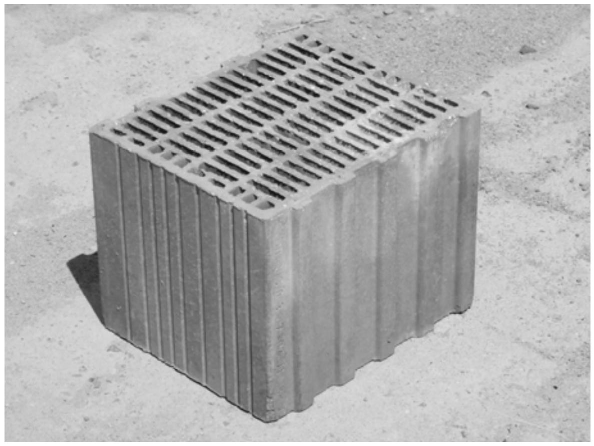
- Only Alice and Bob knows K_a (or K_b)

- Eve has access to E, D and the Communication Channel but does not know the key K_a (or K_b)

Types of symmetric key ciphers

- Block Ciphers: Symmetric key ciphers, where a block of data is encrypted
- Stream Ciphers: Symmetric key ciphers, where block size=1

Block Ciphers



Block Cipher

- A symmetric key modern cipher encrypts an n bit block of plaintext or decrypts an n bit block of ciphertext.
- Padding:
  - If the message has fewer than n bits, padding must be done to make it n bits.
  - If the message size is not a multiple of n, then it should be divided into n bit blocks and the last block should be padded.

Full Size Key Ciphers

- Transposition Ciphers:
  - Involves rearrangement of bits, without changing value.
  - Consider an n bit cipher
  - How many such rearrangements are possible?
    - n!
  - How many key bits are necessary?
    - $\text{ceil}[\log_2 (n!)]$

Full Size Key Ciphers

- Substitution Ciphers:
  - It does not transpose bits, but substitutes values
  - Can we model this as a permutation?
  - *Yes. The n bit inputs and outputs can be represented as 2^n bit sequences, with one 1 and the rest 0's. This can be thus modeled as a transposition.*
  - Thus it is a permutation of 2^n values, thus needs $\text{ceil}[\log_2(2^n!)]$ bits.

Examples

- **Consider a 3-bit block ciphers. How many bits are needed for the full-size key?**
  - **Transposition cipher: ceil($\log_2 6$)=3 bits.**
  - **Substitution cipher:**
    - **There are 8!=40,320 possible substitutions**
    - **Thus there are ceil($\log_2$(40,320))=16 bits**
  - **Lots of unused key.**

Permutation Group

- The fact that the full-size key transposition or substitution cipher is a permutation shows cascading is not of use.
- This is because permutation forms a group under the composition operation.
- Multiple applications of the ciphers has the same effect as a single application of the transformation.

Partial-Size Key Ciphers

- Actual ciphers cannot use full size keys, as the size is large.
- Block ciphers are substitution ciphers (and not transpositions). Why?
- Consider DES, with 64 bit block cipher.
  - Size of full key= $ceil(\log_2(2^{64}!))\approx 2^{70}$
  - Much large compared to 56 bits which is actually used.

Is the partial-key cipher a group?

- Important, because if yes then again multiple applications of the cipher is useless.
- A partial-key cipher is a group if it is a subgroup of the corresponding full key cipher.
- It has been proved that the multi-stage DES with a 56 bit key is not a group because no subgroup with 2^{56} mappings can be created from the corresponding group with $2^{64}!$ mappings

Components of a Modern Block Cipher

- Most important components:
  - PBox: It is a key-less fixed transposition cipher
  - SBox: It is a key-less fixed substitution cipher
- They are used to provide:
  - **Diffusion:** it hides the relationship between the ciphertext and the plaintext
  - **Confusion:** it hides the relationship between the ciphertext and the key

Principle of Confusion and Diffusion

- The design principles of Block Cipher depends on these properties
- The S-Box is used to provide **confusion**, as it is dependent on the unknown key
- The P-Box is fixed, and there is no confusion due to it
- But it provides **diffusion**
- Properly combining these is necessary.

Diffusion (P) Boxes

- Straight Boxes

Example 24x24 Box

01	15	02	13	06	17	03	19	09	04	21	11
14	05	12	16	18	07	24	10	23	08	22	20

- Expansion Boxes

Example 12x24 Box

01	03	02	01	06	17	03	07	09	04	09	11
02	05	12	04	06	07	12	10	11	08	10	08

- Compression Boxes

Example 24x12 Box

01	15	02	13	06	17	03	19	09	04	21	11

SBox

An SBox (substitution box) is an mxn substitution box, where m and n are not necessarily same.

Each output bit is a Boolean function of the inputs.

$$y_1 = f_1(x_1, x_2, ..., x_n)$$
$$y_2 = f_2(x_1, x_2, ..., x_n)$$
$$...$$
$$y_m = f_m(x_1, x_2, ..., x_n)$$

Non-linear SBox

$$y_1 = a_{11}x_1 \oplus a_{12}x_2 \oplus \ldots \oplus a_{1n}x_n$$
$$y_2 = a_{21}x_1 \oplus a_{22}x_2 \oplus \ldots \oplus a_{2n}x_n$$
$$\ldots$$
$$y_m = a_{m1}x_1 \oplus a_{m2}x_2 \oplus \ldots \oplus a_{mn}x_n$$

In a non-linear S-Box, each of the elements cannot be expressed as above.

Eg.

$$y_1 = x_1x_3 \oplus x_2, y_2 = x_1x_2 \oplus x_3$$

Other Components

- Circular Shift:
  - It shifts each bit in an n-bit word k positions to the left. The leftmost k bits become the rightmost bits.
  - Invertible Transformation
- Swap:
  - A special type of shift operation where k=n/2
- Other operations involve split and combine.
- An important component is exclusive-or operation

Properties of Exor

Ex-or is a binary operator, which results in 1 when both the inputs have a different logic. Otherwise, it computes 0.

Symbol: $\oplus$

Closure: Result of exoring two n bit numbers is also n bits.

Associativity: Allows to use more than one '$\oplus$'s in any order:

$$x \oplus (y \oplus z) = (x \oplus y) \oplus z$$

Commutavity: $x \oplus y = y \oplus x$

Identity: The identity element is the n bit 0, represented by

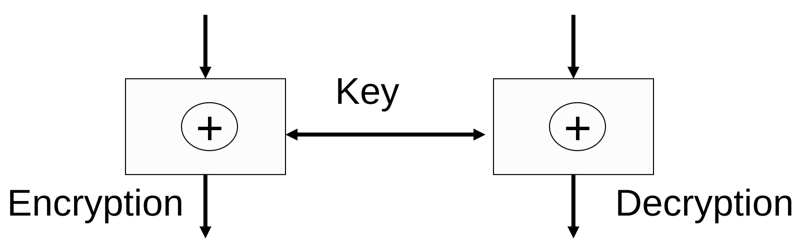
$(00...0)=0^n$

Thus, $x \oplus 0^n = x$

Inverse: Each word is the additive inverse of itself.

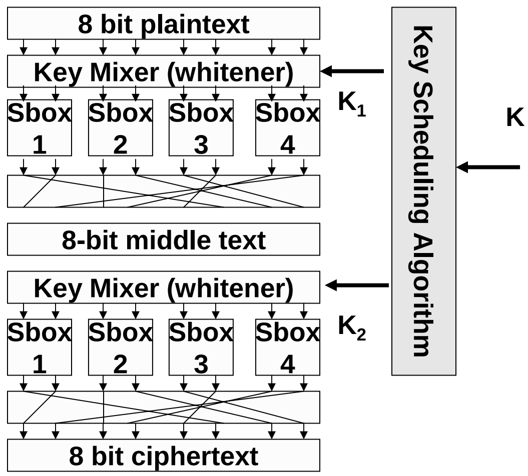
Thus, $x \oplus x = 0^n$

Application of Ex-or

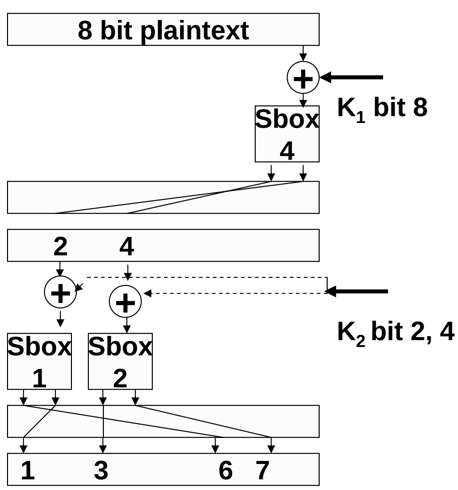


- **The key is known to both the encryptor and decryptor and helps to recover the plaintext.**

A product cipher made of 2 rounds



Diffusion and Confusion



Practical Ciphers

- Large data blocks
- More S-Boxes
- More rounds
- These help to improve the diffusion and confusion in the cipher.

Two classes of product ciphers

- Feistel Ciphers, example DES (Data Encryption Standard)
- Non-Feistel Ciphers (Substitution Permutation Networks), example AES (Advanced Encryption System)

Feistel Cipher

- **Feistel cipher** refers to a type of block cipher design, not a specific cipher
- Split plaintext block into left and right halves: Plaintext = (L_0,R_0)
- For each round $i=1,2,...,n$, compute

  $\mathbf{L_i= R_{i-1}}$

  $\mathbf{R_i= L_{i-1} \oplus f(R_{i-1},K_i)}$

  where f is **round function** and K_i is **subkey**
- Ciphertext = (L_n,R_n)

Feistel Permutation

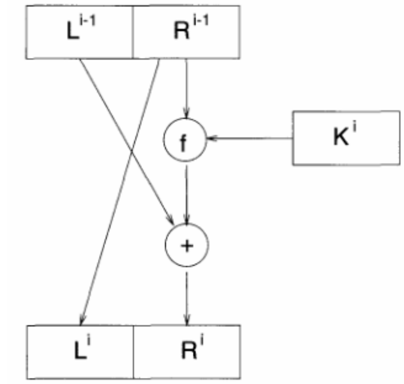
- Decryption: Ciphertext = (L_n,R_n)
- For each round $i=n,n-1,…,1$, compute

  $\mathbf{R_{i-1} = L_i}$

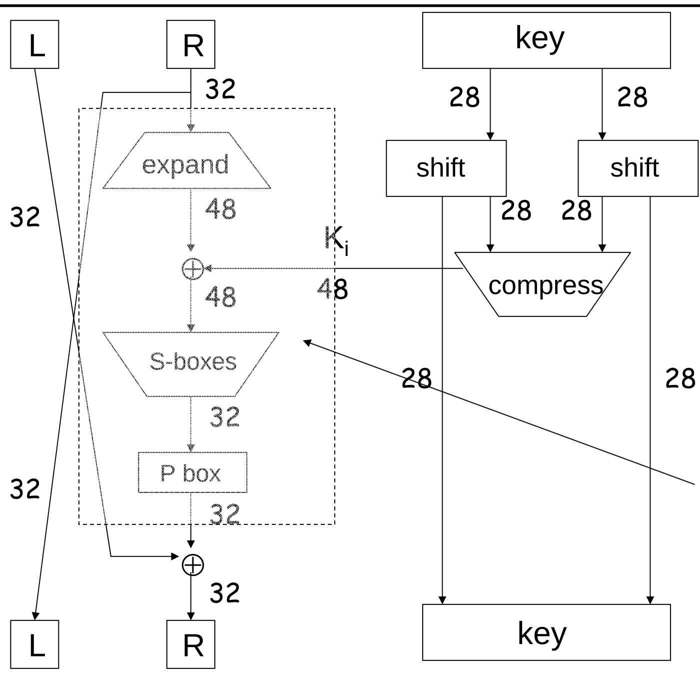
  $\mathbf{L_{i-1} = R_i \oplus f(R_{i-1},K_i)}$

  where f is round function and K_i is subkey
- Plaintext = (L_0,R_0)
- Formula "works" for any function F
- But only secure for certain functions F

Encryption



Repeating/ Iterating this transformation we obtain the Feistel Cipher



One Round of DES

Function f

Note that the design of DES is reduced to the design of f, which works on shorter lengths

Non-Feistel Ciphers

- Composed of only invertible components.
- Input to round function consists of key and the output of previous round
- These functions are obtained by the repeated application of Substitution (invertible SBoxes) and Permutation.
- Thus they are called Substitution Permutation Networks (SPN).

Further Reading

- C. E. Shannon, *Communication Theory of Secrecy Systems.* Bell Systems Technical Journal, 28(1949), 656-715
- B. A Forouzan, *Cryptography & Network Security, Tata Mc Graw Hills, Chapter 5*
- Douglas Stinson, *Cryptography Theory and Practice, 2nd Edition*, Chapman & Hall/CRC

Points to ponder!

- State true or false:
  - *The following key mixing technique is linear wrt. exclusive-or:*
    - *$y=(x + k) \bmod 2^8$, where x and k are 8 bit numbers, and '+' denotes integer addition.*
  - *Having a final permutation step in an SPN (Substitution Permutation Network) cipher has no effect on the security of a block cipher.*

Next Day's Topic

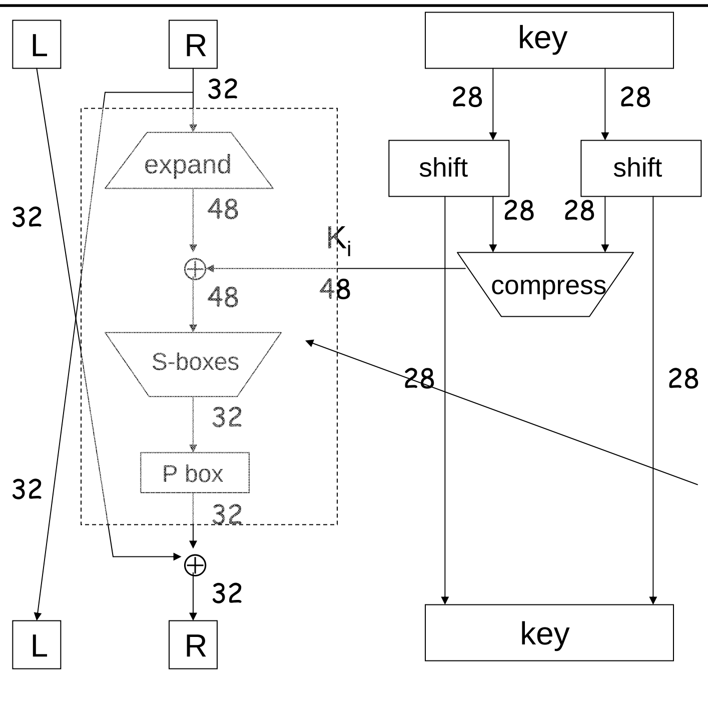
- Designs of Modern Block Ciphers:
  - Data Encryption Standard (DES)
  - Advanced Encryption Standard (AES)

Data Encryption Standard

- DES developed in 1970's
- Based on IBM Lucifer cipher
- U.S. government standard
- DES development was controversial
  - NSA was secretly involved
  - Design process not open
  - Key length was reduced
  - Subtle changes to Lucifer algorithm

DES Numerology

- DES is a Feistel cipher
- 64 bit block length
- 56 bit key length
- 16 rounds
- 48 bits of key used each round (subkey)
- Each round is simple (for a block cipher)
- Security depends primarily on "S-boxes"
- Each S-boxes maps 6 bits to 4 bits



One Round of DES

Note that the design of DES is reduced to the design of f, which works on shorter lengths

DES Expansion

- Input 32 bits

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15
16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31

- Output 48 bits

31 0 1 2 3 4 3 4 5 6 7 8
7 8 9 10 11 12 11 12 13 14 15 16
15 16 17 18 19 20 19 20 21 22 23 24
23 24 25 26 27 28 27 28 29 30 31 0

DES S-box (Substitution Box)

- 8 “substitution boxes” or S-boxes
- Each S-box maps 6 bits to 4 bits
- S-box number 1

input bits (0,5)

↓ input bits (1,2,3,4)

	0000	0001	0010	0011	0100	0101	0110	0111	1000	1001	1010	1011	1100	1101	1110	1111
00	1110	0100	1101	0001	0010	1111	1011	1000	0011	1010	0110	1100	0101	1001	0000	0111
01	0000	1111	0111	0100	1110	0010	1101	0001	1010	0110	1100	1011	1001	0101	0011	1000
10	0100	0001	1110	1000	1101	0110	0010	1011	1111	1100	1001	0111	0011	1010	0101	0000
11	1111	1100	1000	0010	0100	1001	0001	0111	0101	1011	0011	1110	1010	0000	0110	1101

For other tables refer to Stinson’s Book

S-Box with Table entries in decimal

Output=13

S_1

14	4	13	1	[illegible]	[illegible]	11	8	3	10	6	12	5	9	0	7
0	15	7	4	[illegible]	2	13	1	10	6	12	11	9	5	3	8
4	1	14	8	13	6	2	11	15	12	9	7	3	10	5	0
15	12	8	2	4	9	1	7	5	11	3	14	10	0	6	13

What is the output if input is 101000?

Row=10=2 **Column=0100=4**

Properties of the S-Box

- There are several properties
- We highlight some:
  - The rows are permutations
  - The inputs are a non-linear combination of the inputs
  - Change one bit of the input, and half of the output bits change **(Avalanche Effect)**
  - Each output bit is dependent on all the input bits

DES P-box (Permutation Box)

- Input 32 bits

  0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15
  16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31

- Output 32 bits

  15 6 19 20 28 11 27 16 0 14 22 25 4 17 30 9
  1 7 23 13 31 26 2 8 18 12 29 5 21 10 3 24

DES Subkey

- 56 bit DES key, 0,1,2,…,55
- Left half key bits, `LK`

49	42	35	28	21	14	7
0	50	43	36	29	22	15
8	1	51	44	37	30	23
16	9	2	52	45	38	31

- Right half key bits, `RK`

55	48	41	34	27	20	13
6	54	47	40	33	26	19
12	5	53	46	39	32	25
18	11	4	24	17	10	3

DES Subkey

- For rounds `i=1,2,....,n`
  - Let LK = (LK circular shift left by r_i)
  - Let RK = (RK circular shift left by r_i)
  - Left half of subkey K_i is of LK bits

13	16	10	23	0	4	2	27	14	5	20	9
22	18	11	3	25	7	15	6	26	19	12	1

  - Right half of subkey K_i is RK bits

12	23	2	8	18	26	1	11	22	16	4	19
15	20	10	27	5	24	17	13	21	7	0	3

DES Subkey

- For rounds 1, 2, 9 and 16 the shift r_i is 1, and in all other rounds r_i is 2
- Bits 8,17,21,24 of LK omitted each round
- Bits 6,9,14,25 of RK omitted each round
- **Compression permutation** yields 48 bit subkey K_i from 56 bits of LK and RK
- **Key schedule** generates subkey

DES Some Points to Ponder

- An initial perm P before round 1
- Halves are swapped after last round
- A final permutation (inverse of P) is applied to (R_{16},L_{16}) to yield ciphertext
- ***None of these serve any security purpose***

Further Reading

- C. E. Shannon, *Communication Theory of Secrecy Systems*. Bell Systems Technical Journal, 28(1949), 656-715
- B. A Forouzan, *Cryptography & Network Security, Tata Mc Graw Hills, Chapter 5*
- Douglas Stinson, *Cryptography Theory and Practice, 2^{nd} Edition*, Chapman & Hall/CRC

Next Day's Topic

- Linear Cryptanalysis of SPN ciphers