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Variants of Classical One-Dimensional Bin Packing

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33.1 Introduction

Few problems compete with the bin packing problem in having fascinated so many people for so long a time. Research into the classical bin packing problem dates back over three decades to the early 1970s. In the original version, a list $L = (a_1, a_2, \ldots, a_n)$ of n items, each with a size no larger than 1, is given along with an infinite supply of unit capacity bins. The goal is to pack the list into as few bins as possible so that no bin capacity is exceeded. Because the problem is NP-hard, most research has concentrated on designing fast approximation algorithms with good performance guarantees. The studies have spanned both online and offline algorithms, and have applied both combinatorics and probabilistic analysis.

In parallel with the development of approximation algorithms for the classical problem, several variants have been proposed and studied. In this and the next chapter we survey some of the results for these variants. Because of the extensive work done in this area, we cannot possibly hope to cover every problem. Rather, we choose some of the representative ones, and hope that they will give the reader a flavor of the richness of the problem area.

In Section 33.2, we survey the variant in which the number of items packed is maximized. In this problem the number of bins, m, is fixed and the goal is to pack a maximum number of pieces into the m bins. This problem was first proposed by Coffman et al. [1] in 1978, and studied more recently by Boyar et al. [2].

Section 33.3 surveys the variant that places a bound on the number of items that can be packed in each bin. This problem is identical to the classical bin packing problem, except that, for given $k > 0$, each bin can contain at most k items. The problem was first proposed and studied by Krause et al. [3] in 1975.

In Section 33.4, we survey dynamic bin packing, in which packings change with time; each item has an arrival and departure time, which define the time interval during which an item occupies a bin. This problem was first studied by Coffman et al. [4] in 1983.

Their second algorithm is the GREEDY LOOK-AHEAD NEXT FIT (GLANF), which is an offline algorithm. With this algorithm there is one open bin at any given moment. GLANF keeps on filling the current bin with items in their original order unless the first piece is a 1-piece or the addition of the next piece will bring the bin's level to be at least 1. For the latter situation GLANF makes some greedy effort in filling the current open bin to the highest possible level. They proved that, without the 1-pieces,

$$\frac{27}{20} \leq R^{\infty}_{GLANF} \leq \frac{3}{2}$$

and with the 1-pieces,

$$R^{\infty}_{GLANF} \geq \frac{3}{2}$$

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