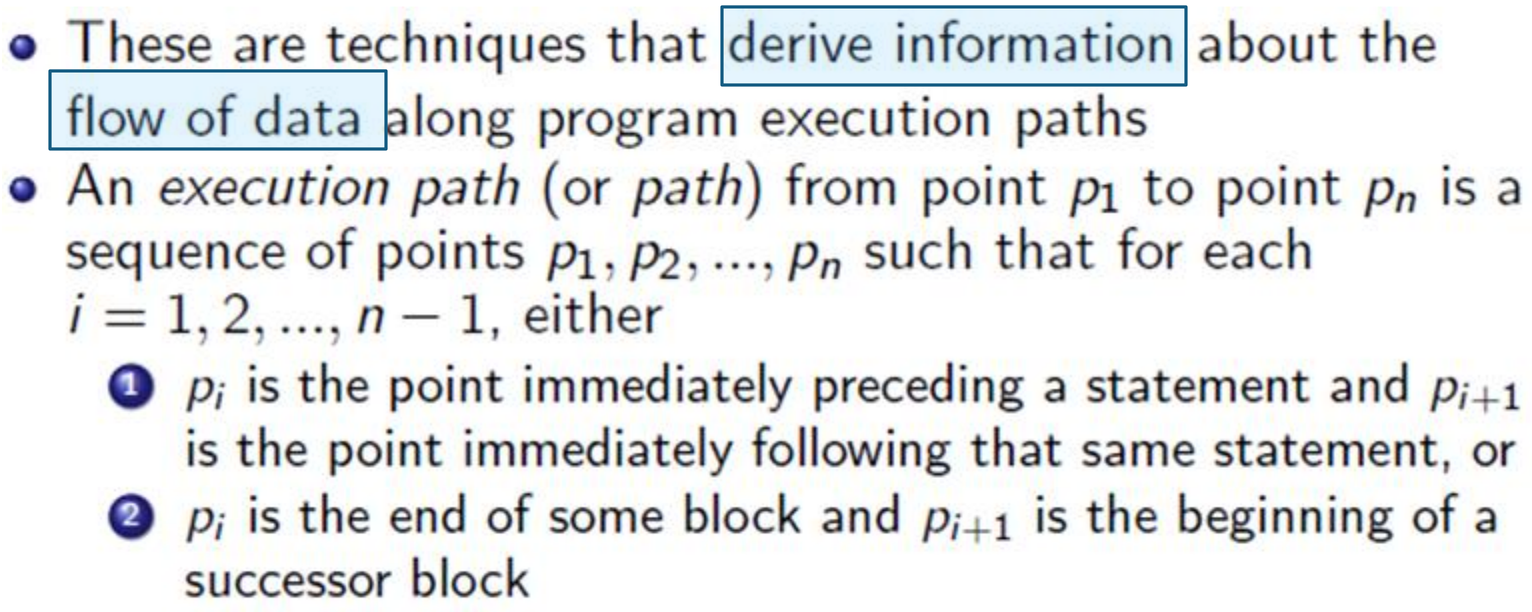
Data Flow Analysis

Data Flow Analysis



- These are techniques that derive information about the flow of data along program execution paths
- An *execution path* (or *path*) from point p_1 to point p_n is a sequence of points $p_1, p_2, \ldots, p_n$ such that for each $i = 1, 2, \ldots, n-1$, either
  1. p_i is the point immediately preceding a statement and p_{i+1} is the point immediately following that same statement, or
  2. p_i is the end of some block and p_{i+1} is the beginning of a successor block

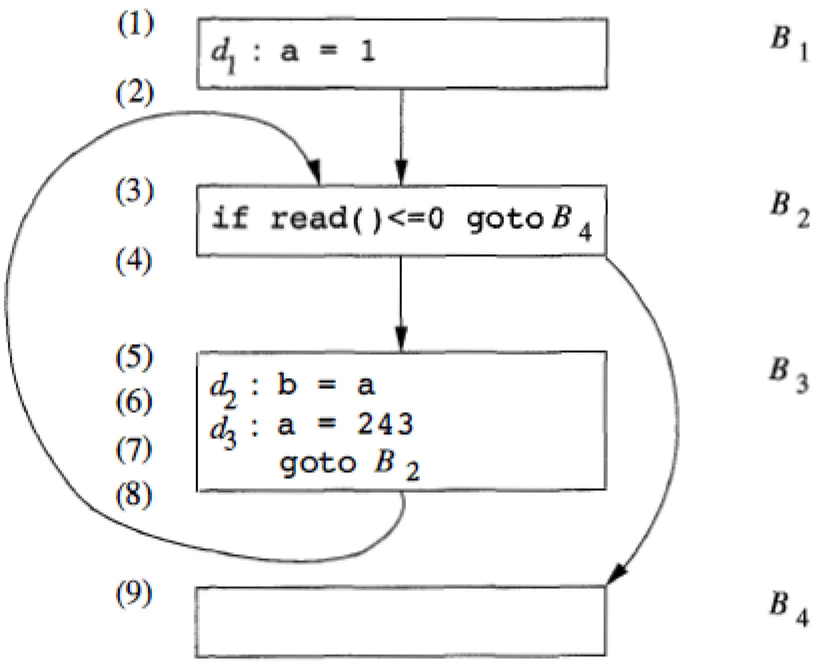
pi

x=y+z

P(i+1)

Data Flow Analysis

Different execution paths of the program



Not entering the loop at all, the shortest complete execution path consists of the program points (1, 2, 3 ,4,9).

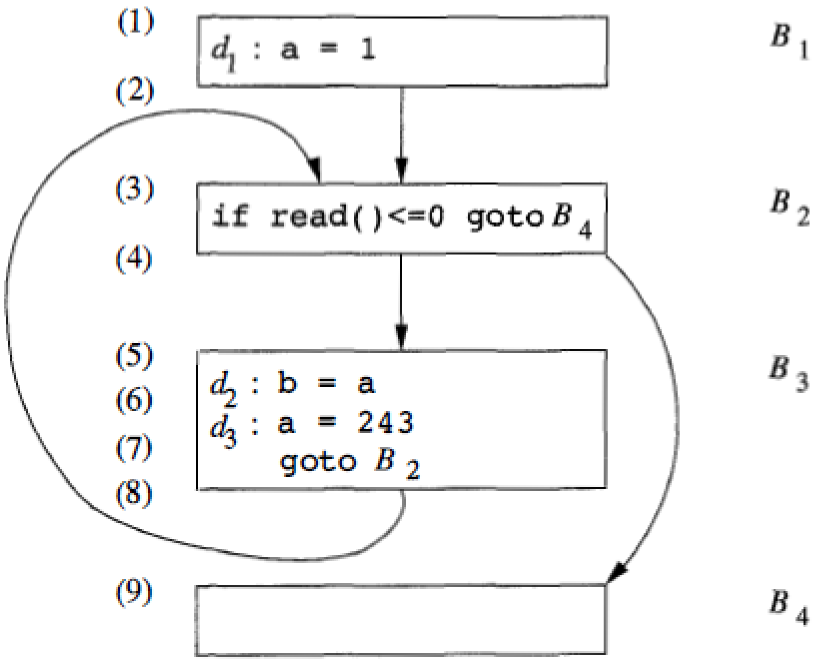
The next shortest path executes one iteration of the loop and consists of the points (1, 2, 3, 4 ,5,6,7,8,3,4,9).

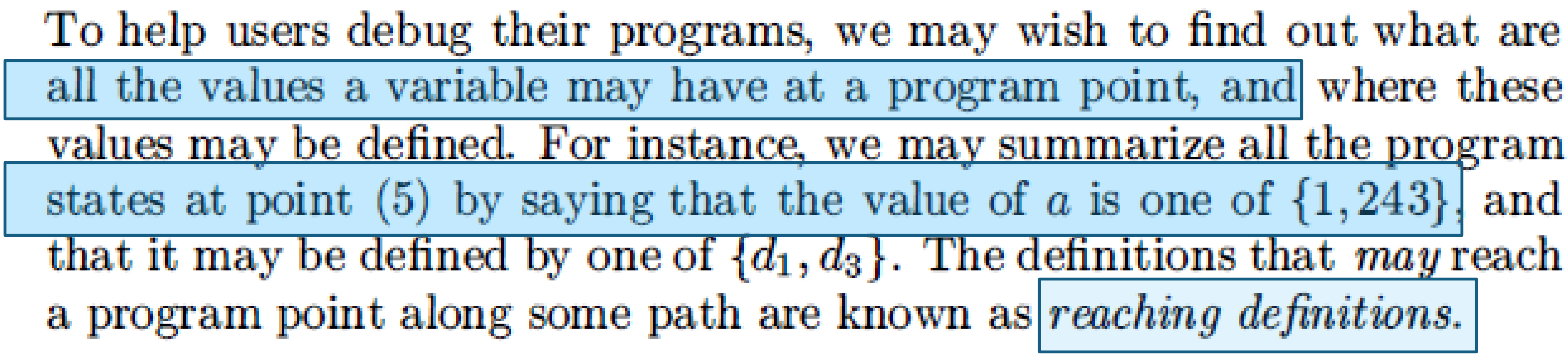
Flow of data value

- For example, the first time program point (5) is executed, the value of a is **1** due to definition d1.
  - We say that **d1 reaches point (5)** in the first iteration.

In subsequent iterations, d3 reaches point (5) and the value of a is 243.

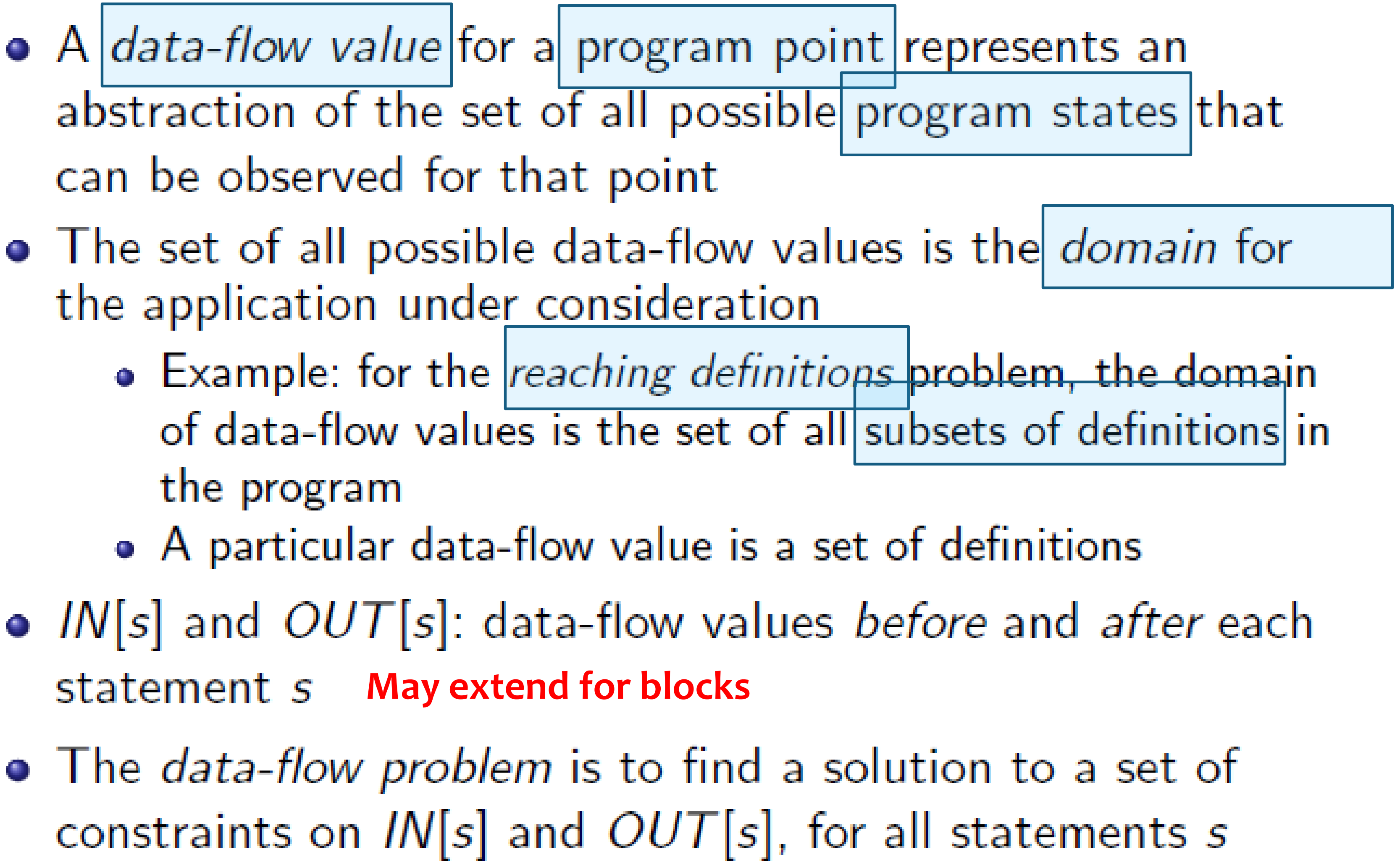
Data Flow Analysis





To help users debug their programs, we may wish to find out what are all the values a variable may have at a program point, and where these values may be defined. For instance, we may summarize all the program states at point (5) by saying that the value of a is one of $\{1, 243\}$, and that it may be defined by one of $\{d_1, d_3\}$. The definitions that *may* reach a program point along some path are known as *reaching definitions*.

DFA Steps



- A *data-flow value* for a program point represents an abstraction of the set of all possible program states that can be observed for that point
- The set of all possible data-flow values is the *domain* for the application under consideration
  - Example: for the *reaching definitions* problem, the domain of data-flow values is the set of all subsets of definitions in the program
  - A particular data-flow value is a set of definitions
- $IN[s]$ and $OUT[s]$: data-flow values *before* and *after* each statement s

May extend for blocks

- The *data-flow problem* is to find a solution to a set of constraints on $IN[s]$ and $OUT[s]$, for all statements s

Data Flow Analysis

- We denote the data-flow values before and after each statement s by IN[S] and OUT[S] , respectively.

- The data-flow problem is to find a solution to a set of constraints on the IN [S] 'S and OUT[S] 'S, for all statements s.

- There are two sets of constraints:
- (a) “Transfer functions” (based on the semantics of the statements)
- (b) Flow of control functions.

(a) Transfer Functions

- The data-flow values before and after a statement are constrained by the **TF** (semantics of the statement)

p(i)
b=a
p(i+1)

- For example, suppose data-flow analysis involves **determining the value** of variables at points.

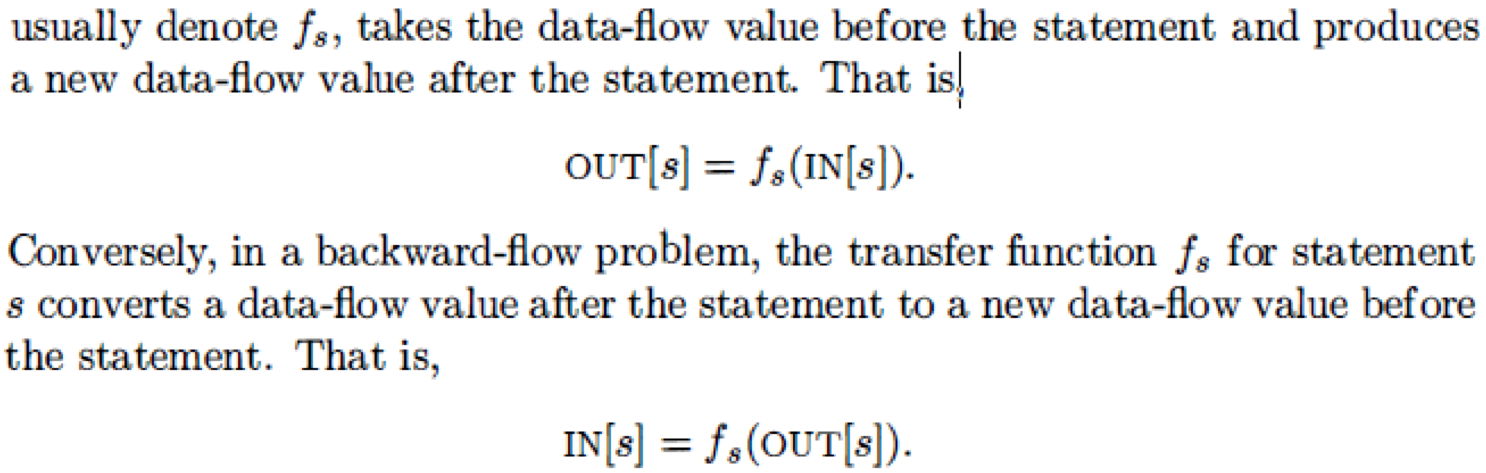
- If variable **a** has value **v** **before executing statement b = a,** then both a and b will have the value **v after the statement**.

- This **relationship** between the data-flow values **before** and **after** the assignment statement is known as a **transfer function.**

(a) Transfer Functions

- Transfer functions come in two flavors: **information may propagate forward** along execution paths,
- Or it may **flow backwards** up the execution paths.
- In **a forward-flow problem,** the **transfer function fs** of a statement **s,**
- (i) **takes** the data-flow value **before the statement** and
- (ii) **produces** a new data-flow value **after the statement**

(a) Transfer Functions



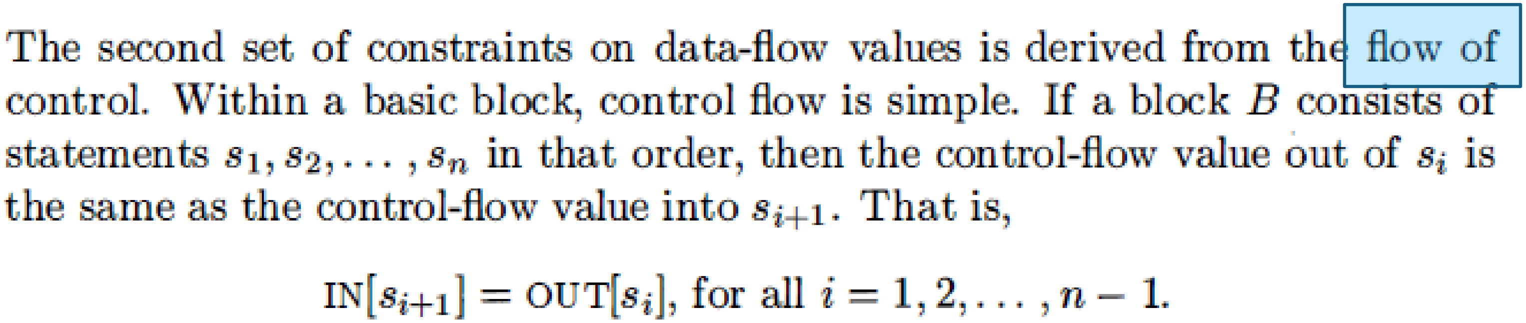
usually denote f_s, takes the data-flow value before the statement and produces a new data-flow value after the statement. That is,

$$\text{OUT}[s] = f_s(\text{IN}[s]).$$

Conversely, in a backward-flow problem, the transfer function f_s for statement s converts a data-flow value after the statement to a new data-flow value before the statement. That is,

$$\text{IN}[s] = f_s(\text{OUT}[s]).$$

(b) Control-Flow Constraints



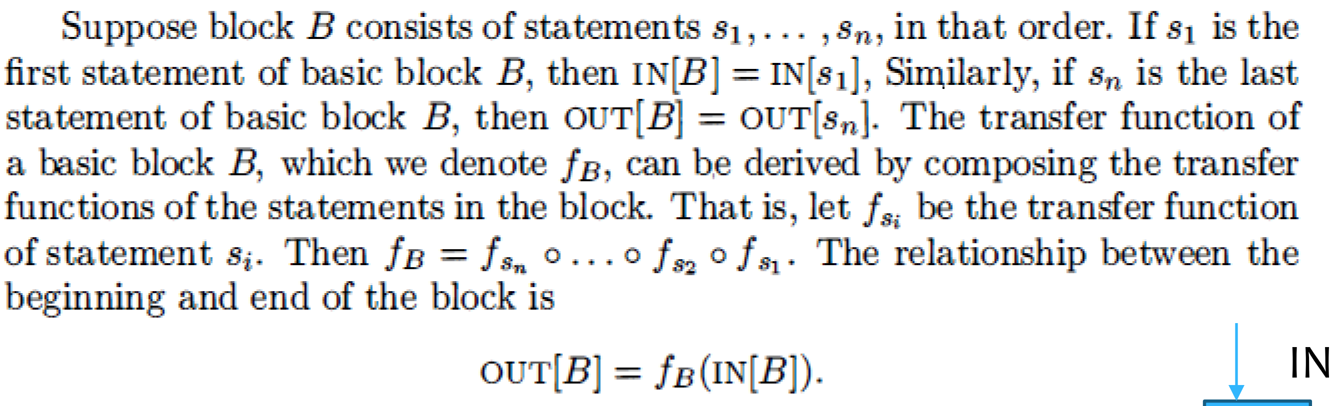
The second set of constraints on data-flow values is derived from the flow of control. Within a basic block, control flow is simple. If a block B consists of statements $s_1, s_2, \ldots, s_n$ in that order, then the control-flow value out of s_i is the same as the control-flow value into s_{i+1}. That is,

$$\text{IN}[s_{i+1}] = \text{OUT}[s_i], \text{ for all } i = 1, 2, \ldots, n-1.$$

- However, **control-flow edges between basic blocks** create more complex constraints between the **last statement of one basic block** and the **first statement of the following block.**

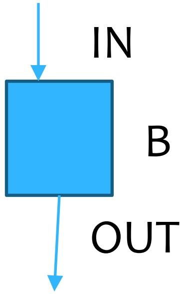
- For example, if we wish to collect **all the definitions that may reach a program point,**
- Then the set of definitions reaching the leader statement of a basic block is the
- **union** of the definitions after the last statements of each of the predecessor blocks.

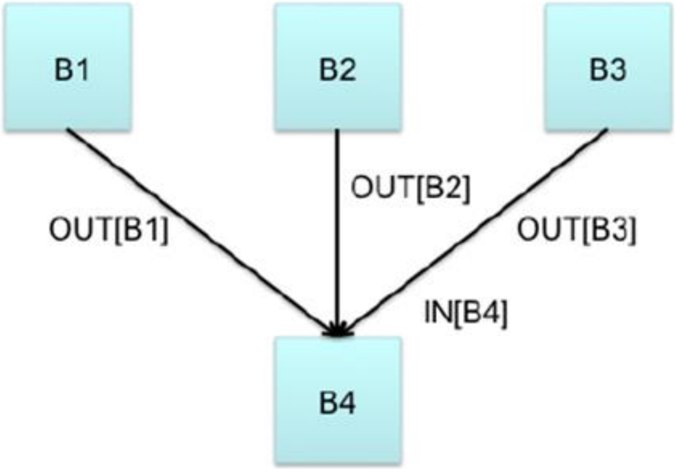
(b) Control-Flow Constraints



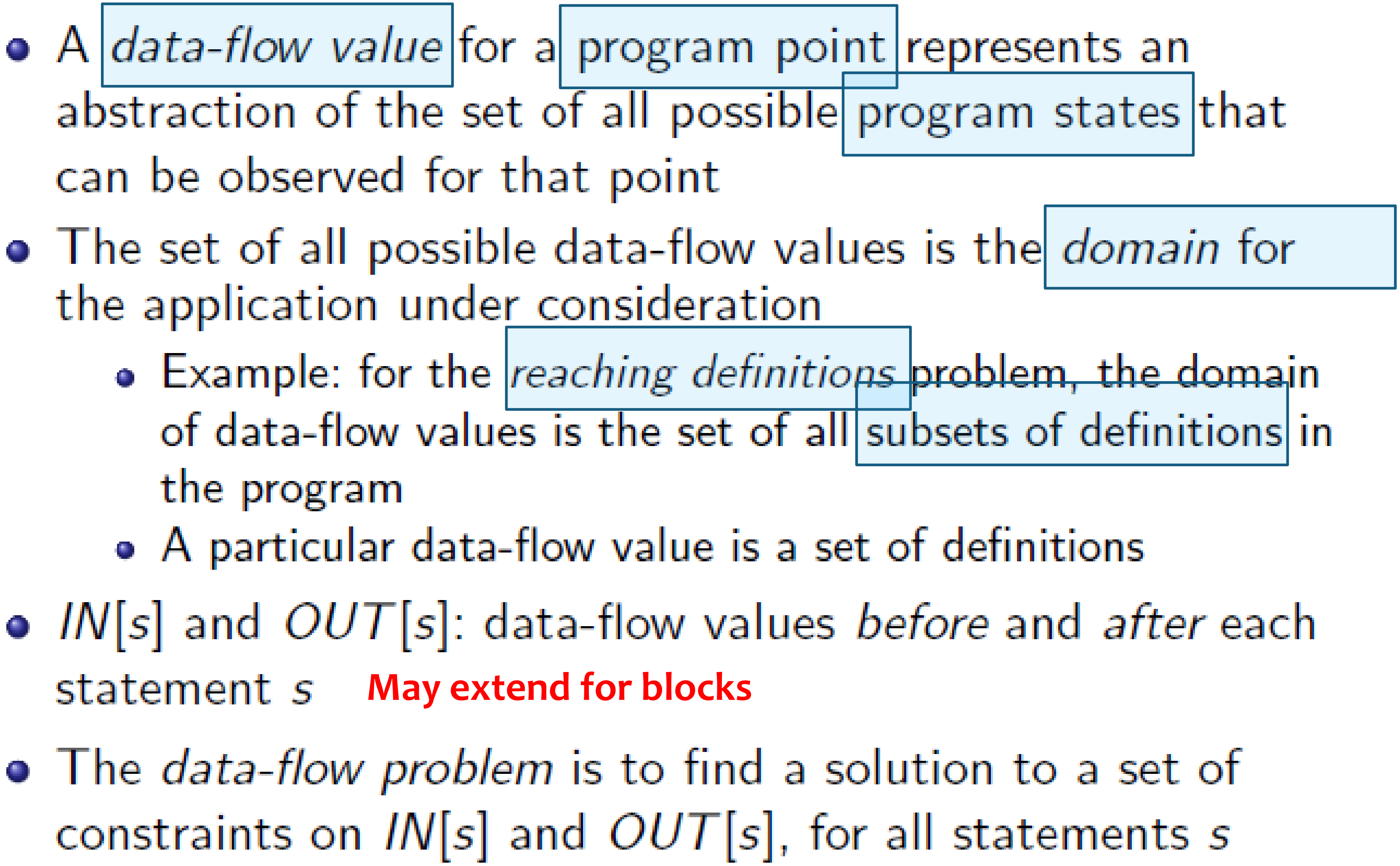
Suppose block B consists of statements $s_1, \ldots, s_n$, in that order. If s_1 is the first statement of basic block B, then $\text{IN}[B] = \text{IN}[s_1]$, Similarly, if s_n is the last statement of basic block B, then $\text{OUT}[B] = \text{OUT}[s_n]$. The transfer function of a basic block B, which we denote f_B, can be derived by composing the transfer functions of the statements in the block. That is, let f_{s_i} be the transfer function of statement s_i. Then $f_B = f_{s_n} \circ \ldots \circ f_{s_2} \circ f_{s_1}$. The relationship between the beginning and end of the block is

$$\text{OUT}[B] = f_B(\text{IN}[B]).$$





DFA Steps

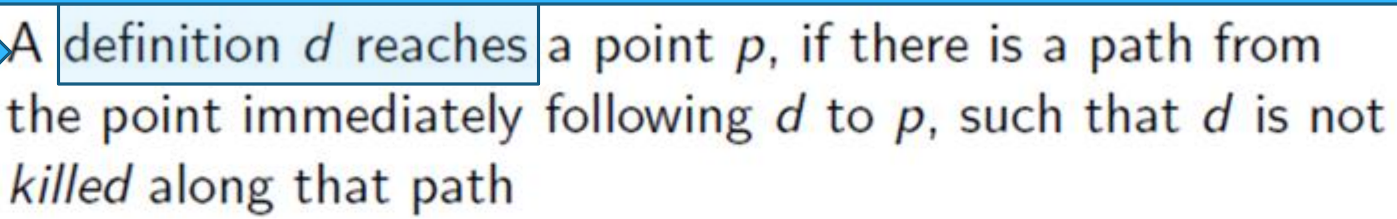


- A *data-flow value* for a program point represents an abstraction of the set of all possible program states that can be observed for that point
- The set of all possible data-flow values is the *domain* for the application under consideration
  - Example: for the *reaching definitions* problem, the domain of data-flow values is the set of all subsets of definitions in the program
  - A particular data-flow value is a set of definitions
- $IN[s]$ and $OUT[s]$: data-flow values *before* and *after* each statement s

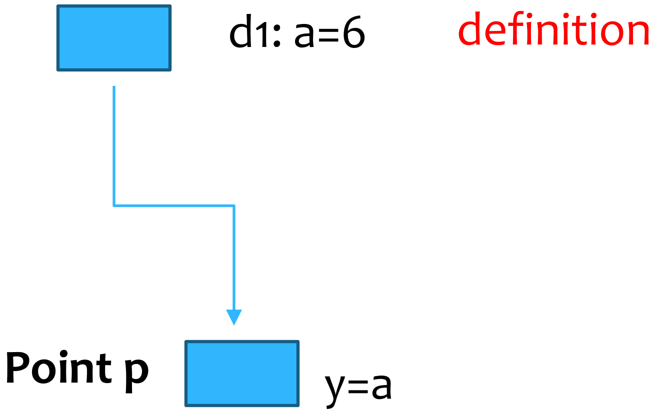
May extend for blocks

- The *data-flow problem* is to find a solution to a set of constraints on $IN[s]$ and $OUT[s]$, for all statements s

Reaching Definitions (RD) Problem



A definition *d* reaches a point *p*, if there is a path from the point immediately following *d* to *p*, such that *d* is not *killed* along that path



d1: a=6

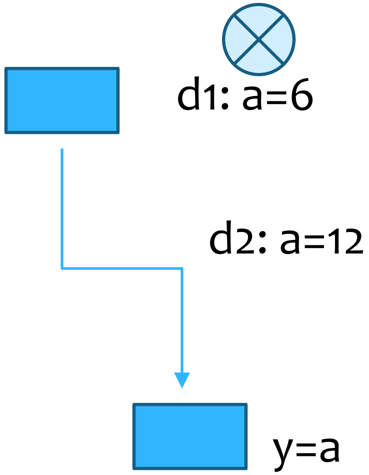
definition

Point p

y=a

Reaching Definitions (RD) Problem

- We *kill* a definition of a variable a, if between two points along the path, there is an assignment to a
- A definition d reaches a point p, if there is a path from the point immediately following d to p, such that d is not *killed* along that path



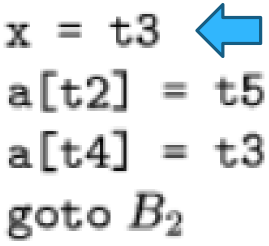
d1: a=6

Kills the definition

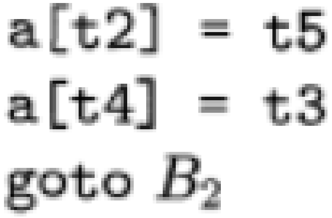
d2: a=12

y=a

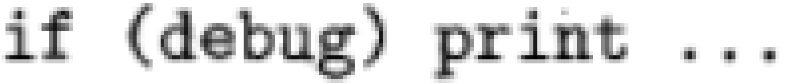
Motivation: Usage



```
x = t3
a[t2] = t5
a[t4] = t3
goto $B_2$
```



```
a[t2] = t5
a[t4] = t3
goto $B_2$
```



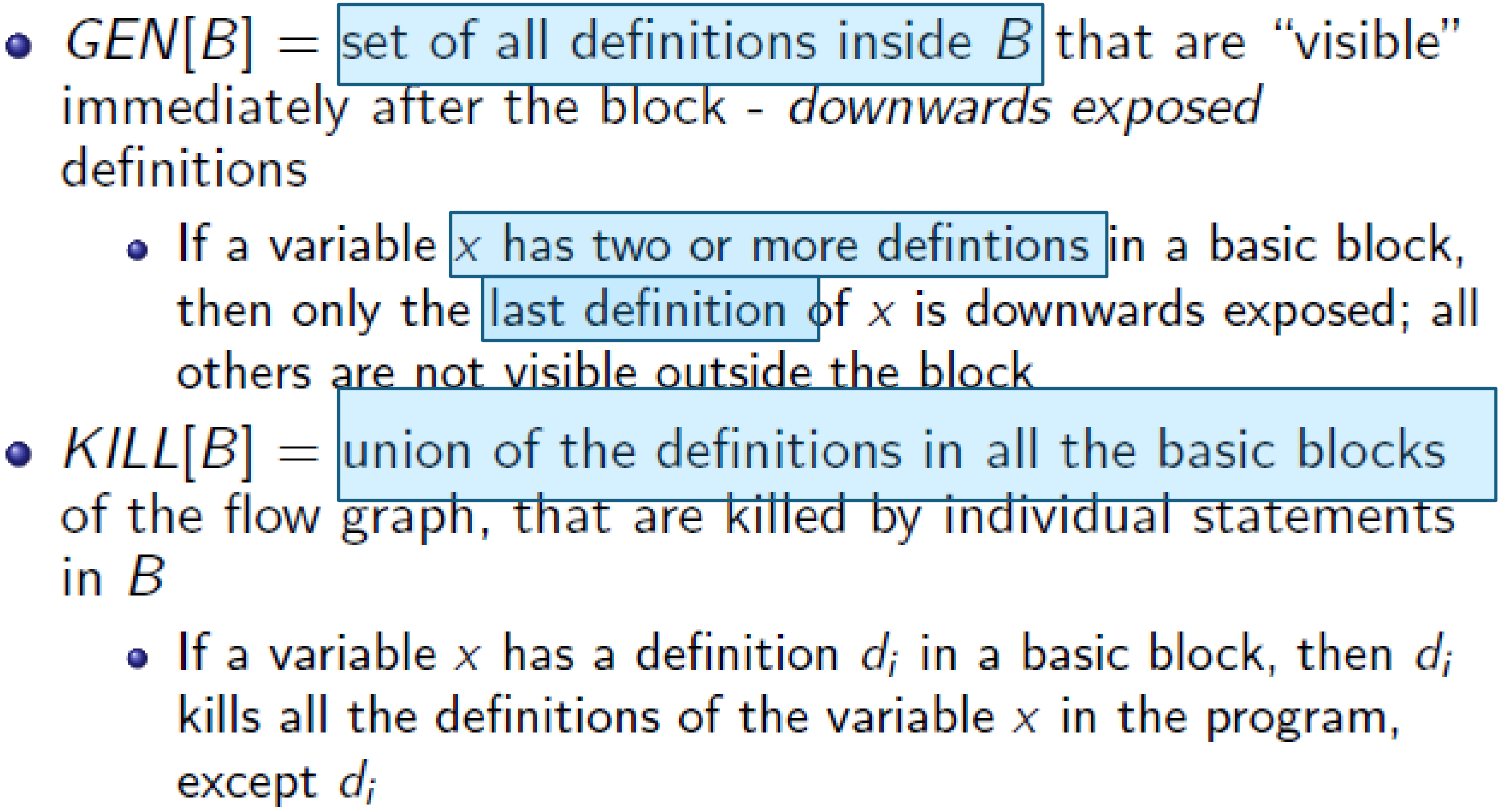
```
if (debug) print ...
```

```
debug = FALSE
```

Copy propagation

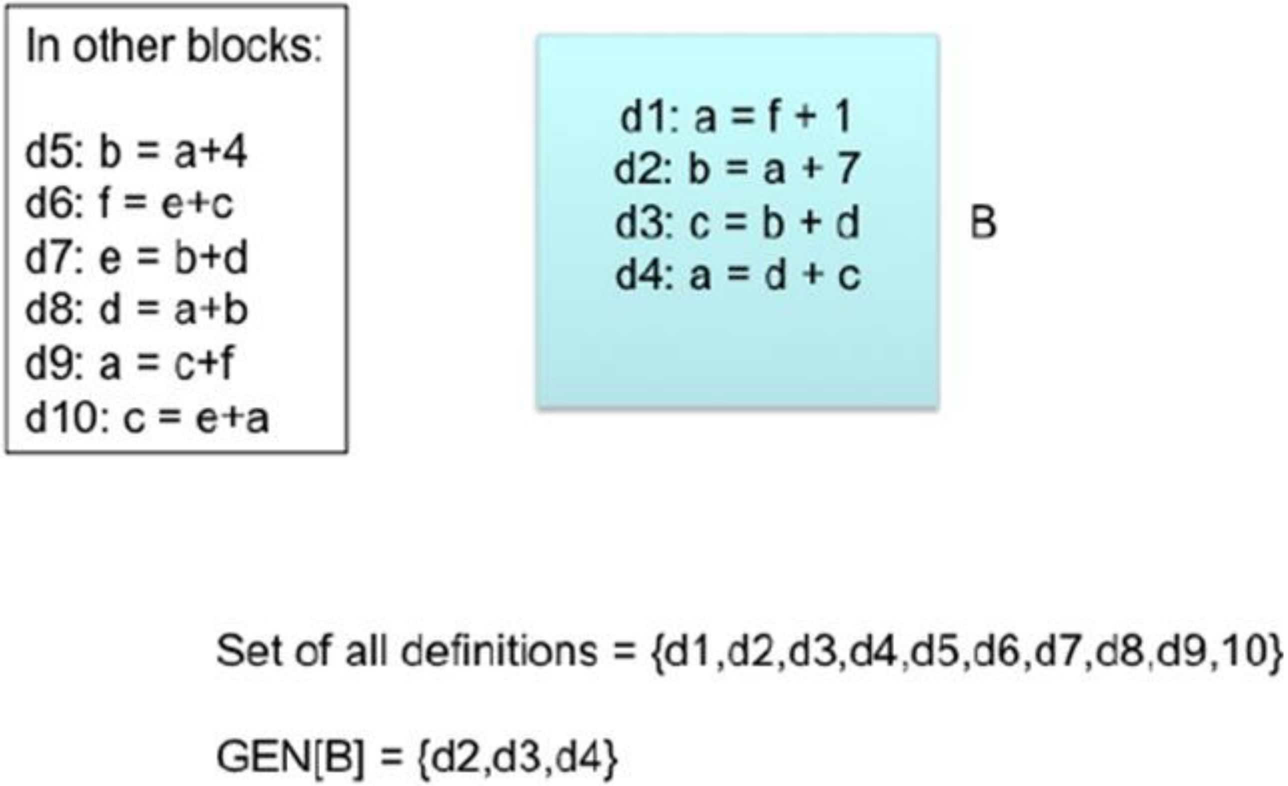
- Drop this code segment
- Constant folding

RD Problem



- $GEN[B]$ = set of all definitions inside B that are "visible" immediately after the block - *downwards exposed* definitions
  - If a variable x has two or more defintions in a basic block, then only the last definition of x is downwards exposed; all others are not visible outside the block
- $KILL[B]$ = union of the definitions in all the basic blocks of the flow graph, that are killed by individual statements in B
  - If a variable x has a definition d_i in a basic block, then d_i kills all the definitions of the variable x in the program, except d_i

RD Analysis: GEN and KILL



In other blocks:

d5: b = a+4
d6: f = e+c
d7: e = b+d
d8: d = a+b
d9: a = c+f
d10: c = e+a

d1: a = f + 1
d2: b = a + 7
d3: c = b + d
d4: a = d + c

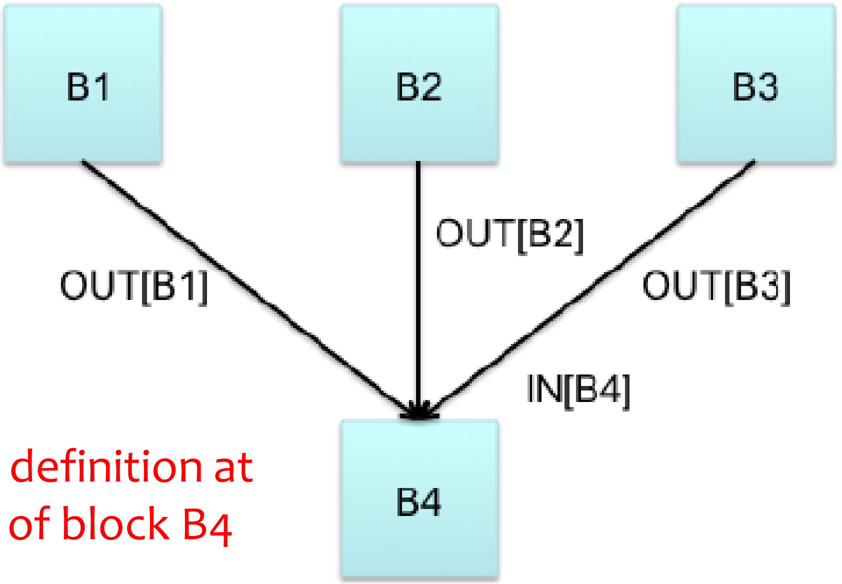
Set of all definitions = {d1,d2,d3,d4,d5,d6,d7,d8,d9,10}

GEN[B] = {d2,d3,d4}

Kills(d9, d5, d10, d1)

RD Analysis: DF Equations

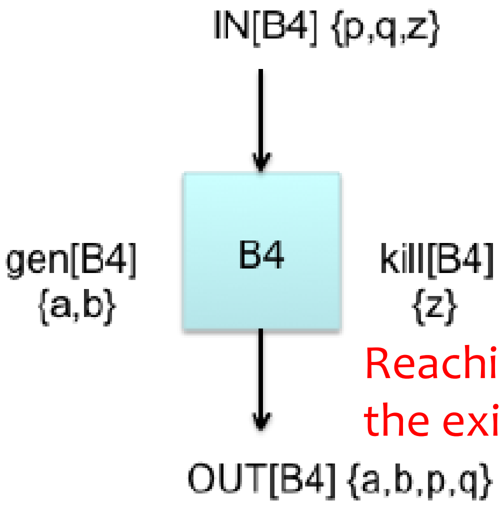
Control flow Eqs



Reaching definition at the entry of block B4

IN[B4] = OUT[B1] U OUT[B2] U OUT[B3]

Transfer Eqs



Reaching definition at the exit of block B4

$$IN[B] = \bigcup_{P \text{ is a predecessor of } B} OUT[P]$$

$$OUT[B] = GEN[B] \bigcup (IN[B] - KILL[B])$$

OUT[B4] = gen[B4] U (IN[B4] – kill[B4])

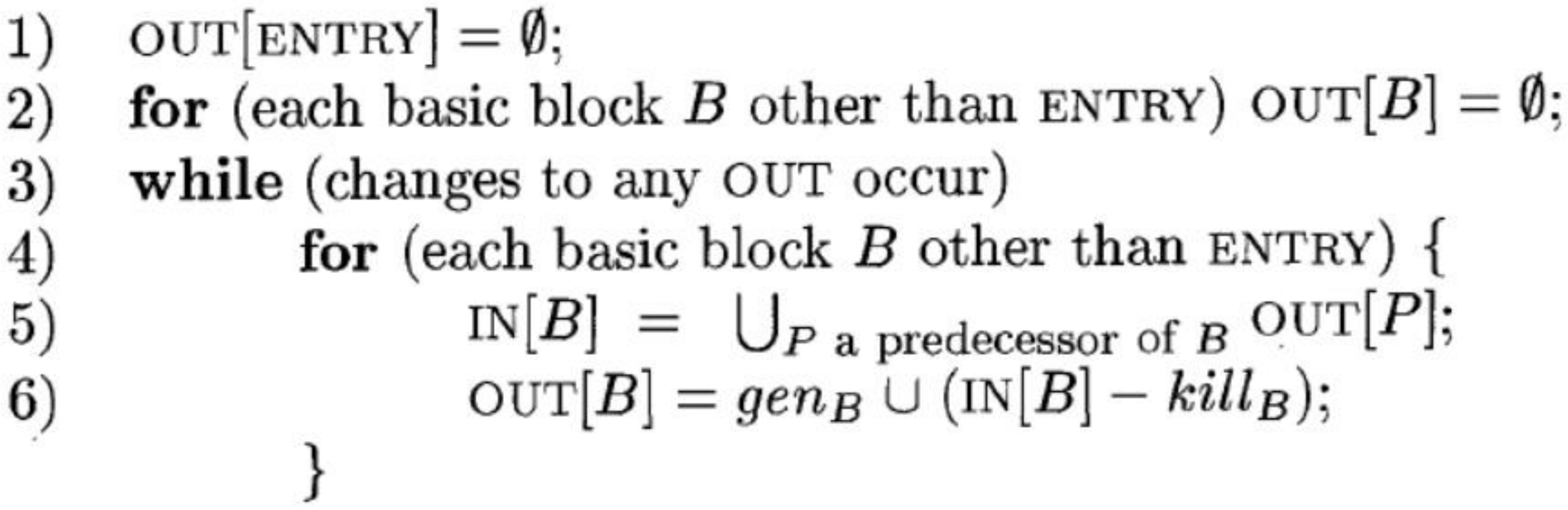
RD Problem

- The data-flow equations (constraints)

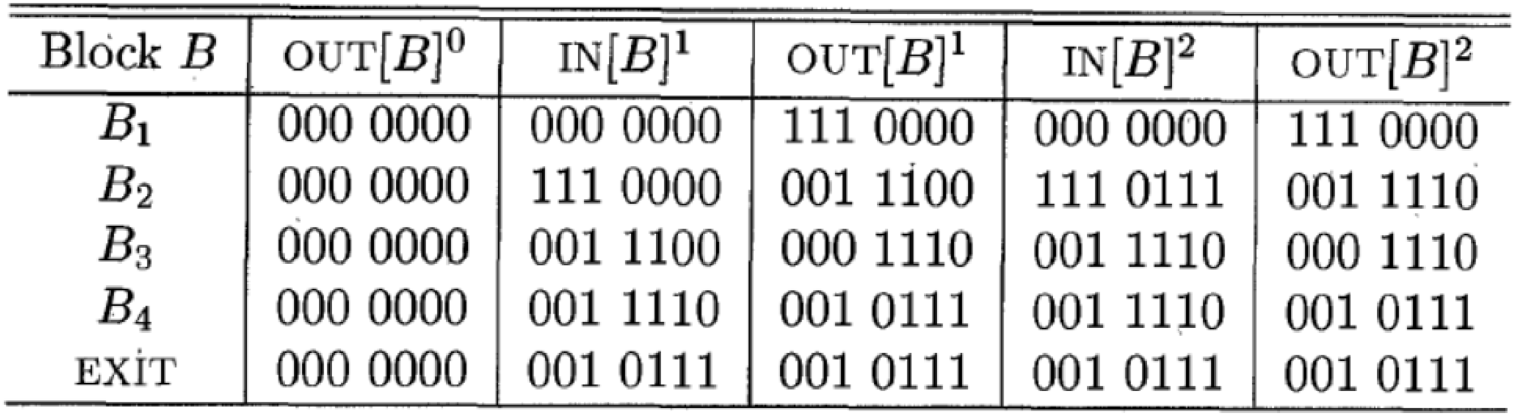
$$
\begin{aligned}
IN[B] &= \bigcup_{P \text{ is a predecessor of } B} OUT[P] \\
OUT[B] &= GEN[B] \bigcup (IN[B] - KILL[B]) \\
IN[B] &= \phi, \text{for all } B \text{ (initialization only)}
\end{aligned}
$$

- If some definitions reach B_1 (entry), then $IN[B_1]$ is initialized to that set
- Forward flow DFA problem (since $OUT[B]$ is expressed in terms of $IN[B]$), confluence operator is $\cup$
  - Direction of flow does not imply traversing the basic blocks in a particular order
  - The final result does not depend on the order of traversal of the basic blocks

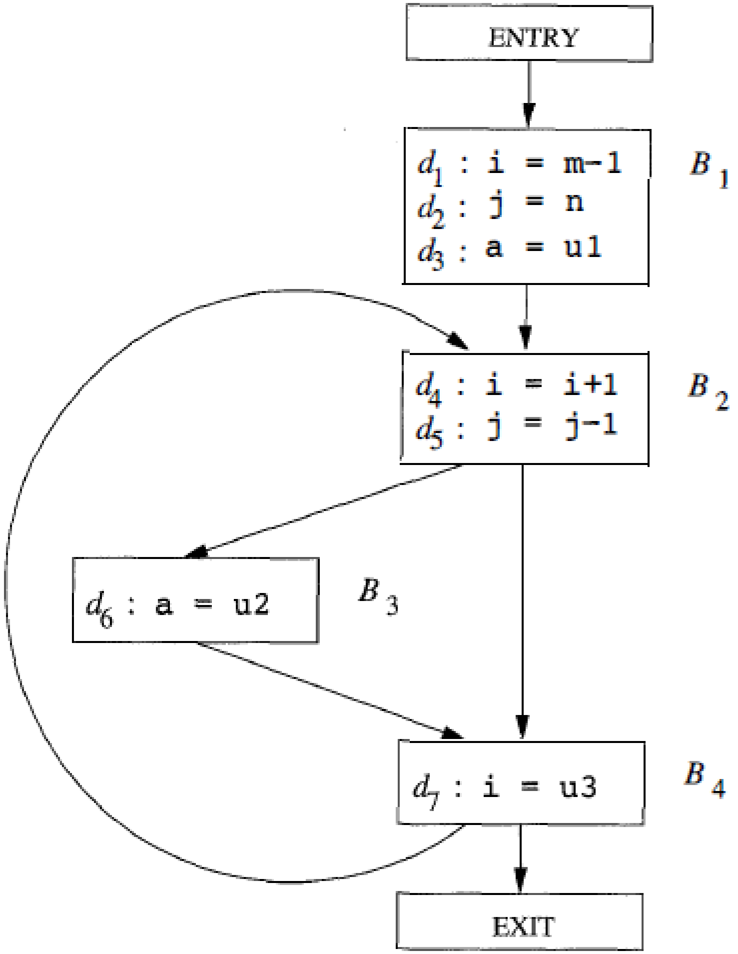
RD algorithm



```
$\text{OUT}[\text{ENTRY}] = \emptyset$;
for (each basic block $B$ other than ENTRY) $\text{OUT}[B] = \emptyset$;
while (changes to any OUT occur)
        for (each basic block $B$ other than ENTRY) {
                $\text{IN}[B] = \bigcup_{P \text{ a predecessor of } B} \text{OUT}[P]$;
                $\text{OUT}[B] = gen_B \cup (\text{IN}[B] - kill_B)$;
        }
```



Block B	$\text{OUT}[B]^0$	$\text{IN}[B]^1$	$\text{OUT}[B]^1$	$\text{IN}[B]^2$	$\text{OUT}[B]^2$
B_1	000 0000	000 0000	111 0000	000 0000	111 0000
B_2	000 0000	111 0000	001 1100	111 0111	001 1110
B_3	000 0000	001 1100	000 1110	001 1110	000 1110
B_4	000 0000	001 1110	001 0111	001 1110	001 0111
EXIT	000 0000	001 0111	001 0111	001 0111	001 0111



d_1 : `i = m-1`
d_2 : `j = n`
d_3 : `a = u1`

d_4 : `i = i+1`
d_5 : `j = j-1`

d_6 : `a = u2`

d_7 : `i = u3`

$gen_{B_1} = \{ d_1, d_2, d_3 \}$

$kill_{B_1} = \{ d_4, d_5, d_6, d_7 \}$

$gen_{B_2} = \{ d_4, d_5 \}$

$kill_{B_2} = \{ d_1, d_2, d_7 \}$

$gen_{B_3} = \{ d_6 \}$

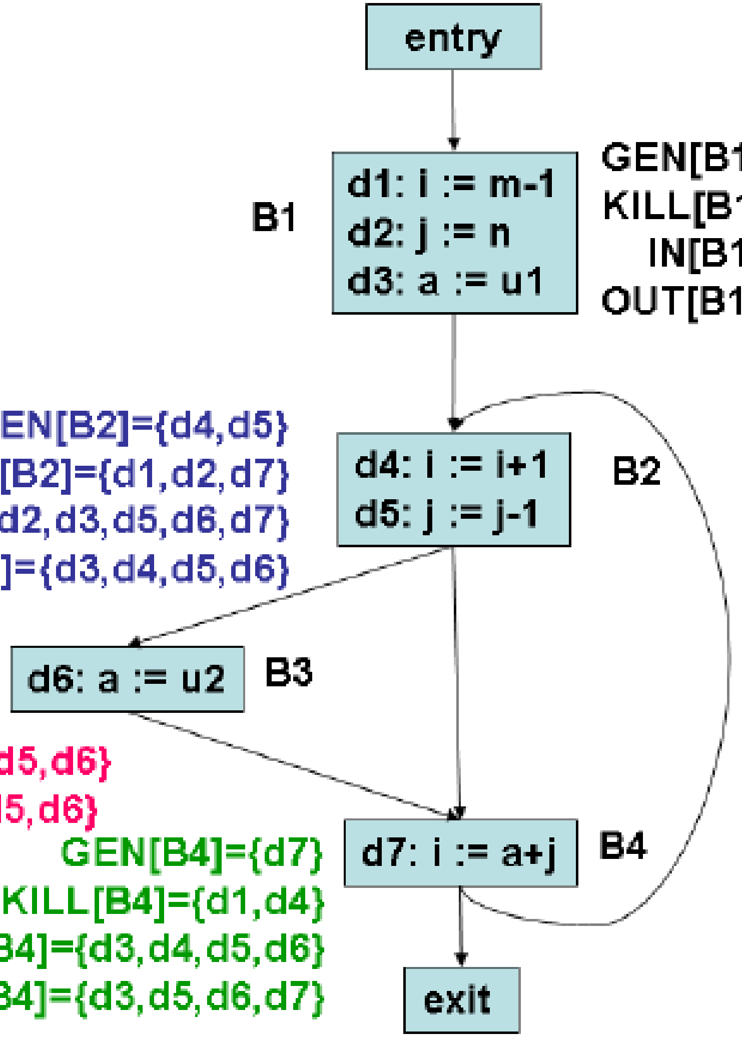
$kill_{B_3} = \{ d_3 \}$

$gen_{B_4} = \{ d_7 \}$

$kill_{B_4} = \{ d_1, d_4 \}$

RD: Bit vector representation

Final dataflow value sets shown in bit vector format



GEN[B1]=	1	1	1	0	0	0	0
KILL[B1]=	0	0	0	1	1	1	1
IN[B1]=	0	0	0	0	0	0	0
OUT[B1]=	1	1	1	0	0	0	0
	d1	d2	d3	d4	d5	d6	d7

GEN[B2]={d4,d5}
KILL[B2]={d1,d2,d7}
IN[B2]={d1,d2,d3,d5,d6,d7}
OUT[B2]={d3,d4,d5,d6}

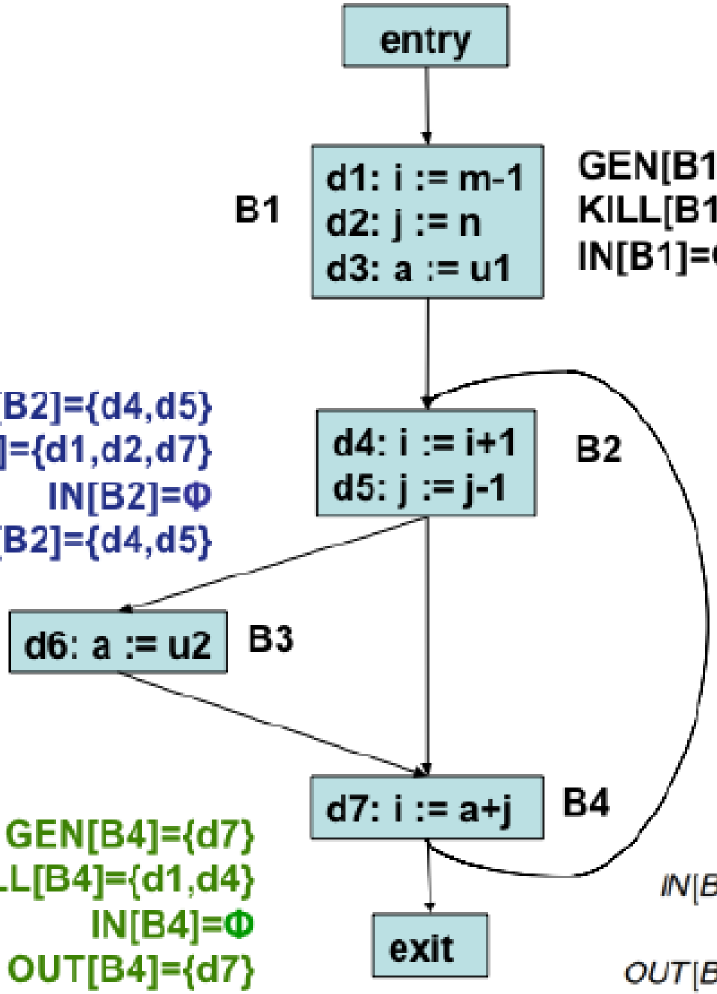
GEN[B3]={d6}
KILL[B3]={d3}
IN[B3]={d3,d4,d5,d6}
OUT[B3]={d4,d5,d6}

GEN[B4]={d7}
KILL[B4]={d1,d4}
IN[B4]={d3,d4,d5,d6}
OUT[B4]={d3,d5,d6,d7}

Adapted from the "Dragon Book", A-W, 1986

RD Analysis: An example

Pass 1



GEN[B1]={d1,d2,d3}
KILL[B1]={d4,d5,d6,d7}
IN[B1]=Φ, OUT[B1]={d1,d2,d3}

GEN[B2]={d4,d5}
KILL[B2]={d1,d2,d7}
IN[B2]=Φ
OUT[B2]={d4,d5}

GEN[B3]={d6}
KILL[B3]={d3}
IN[B3]=Φ
OUT[B3]={d6}

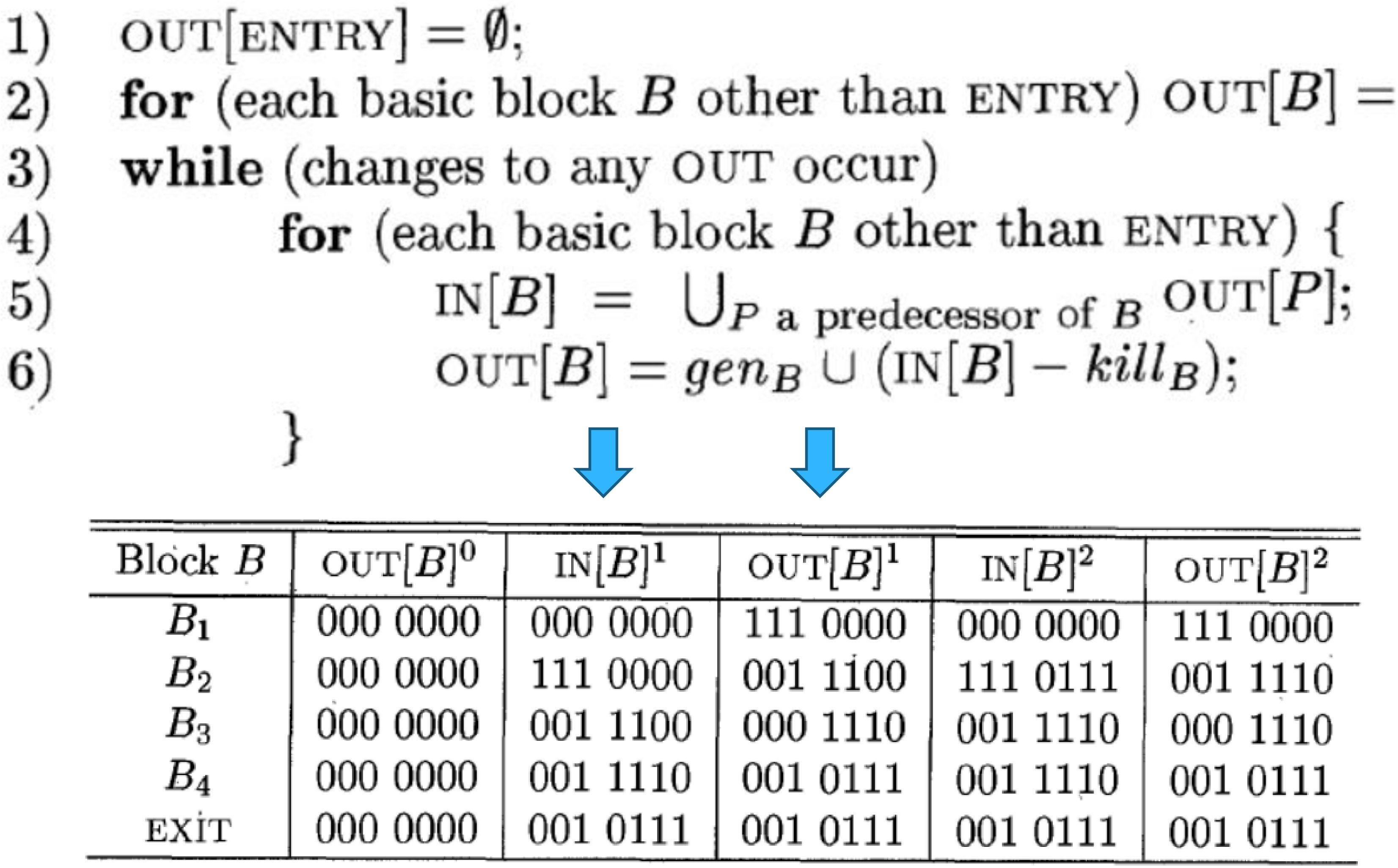
GEN[B4]={d7}
KILL[B4]={d1,d4}
IN[B4]=Φ
OUT[B4]={d7}

Adapted from the "Dragon Book", A-W, 1986

$$IN[B] = \bigcup_{P \text{ is a predecessor of } B} OUT[P]$$

$$OUT[B] = GEN[B] \bigcup (IN[B] - KILL[B])$$

RD algorithm



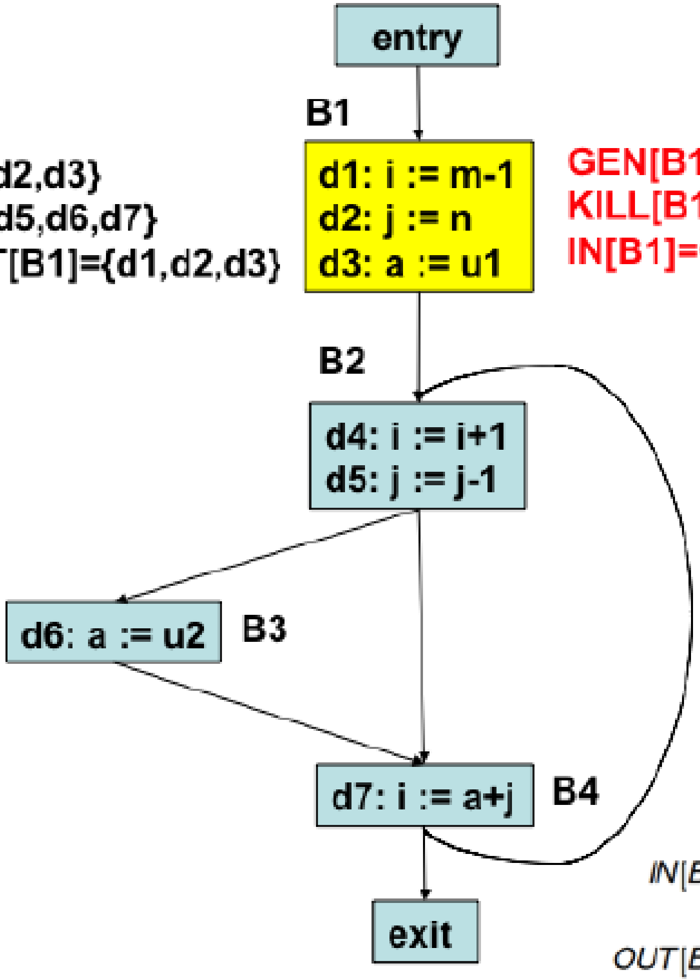
```
OUT[ENTRY] = $\emptyset$;
for (each basic block $B$ other than ENTRY) OUT[$B$] = $\emptyset$;
while (changes to any OUT occur)
    for (each basic block $B$ other than ENTRY) {
        IN[$B$] = $\bigcup_{P \text{ a predecessor of } B}$ OUT[$P$];
        OUT[$B$] = $gen_B \cup (\text{IN}[B] - kill_B)$;
    }
```

Block B	OUT$[B]^0$	IN$[B]^1$	OUT$[B]^1$	IN$[B]^2$	OUT$[B]^2$
B_1	000 0000	000 0000	111 0000	000 0000	111 0000
B_2	000 0000	111 0000	001 1100	111 0111	001 1110
B_3	000 0000	001 1100	000 1110	001 1110	000 1110
B_4	000 0000	001 1110	001 0111	001 1110	001 0111
EXIT	000 0000	001 0111	001 0111	001 0111	001 0111

RD Analysis: An example

Pass 2

GEN[B1]={d1,d2,d3}
KILL[B1]={d4,d5,d6,d7}
IN[B1]=Φ, OUT[B1]={d1,d2,d3}



GEN[B1]={d1,d2,d3}
KILL[B1]={d4,d5,d6,d7}
IN[B1]=Φ, OUT[B1]={d1,d2,d3}

$$IN[B] = \bigcup_{P \text{ is a predecessor of } B} OUT[P]$$

$$OUT[B] = GEN[B] \bigcup (IN[B] - KILL[B])$$

RD algorithm

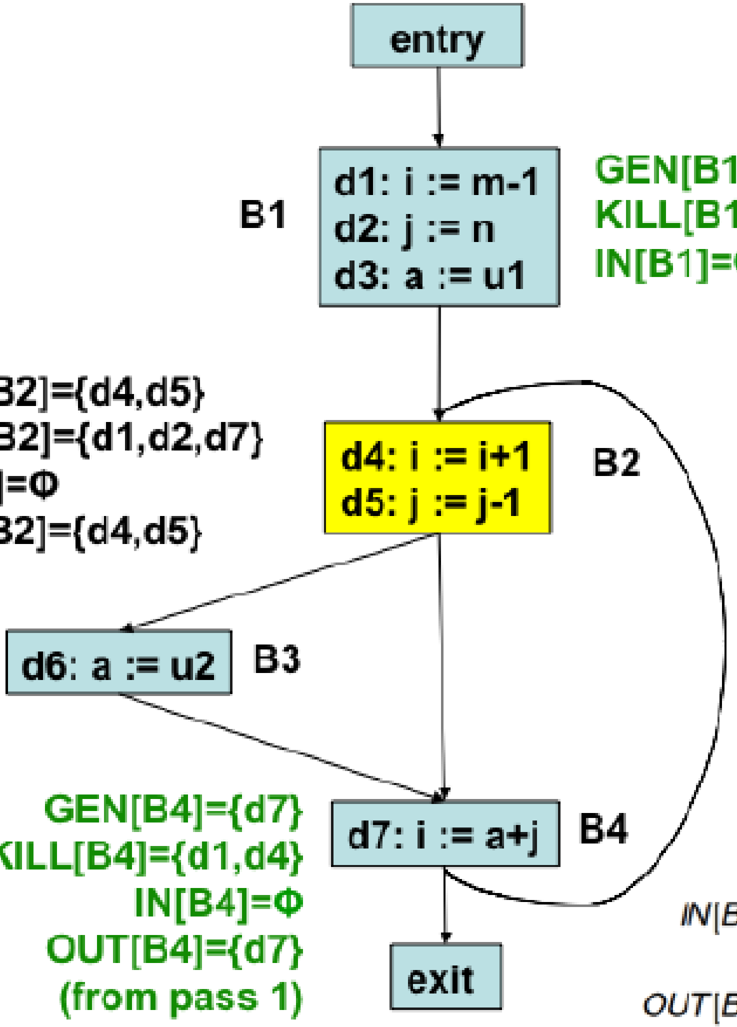
```
$\text{OUT}[\text{ENTRY}] = \emptyset$;
for (each basic block $B$ other than ENTRY) $\text{OUT}[B] = \emptyset$;
while (changes to any OUT occur)
        for (each basic block $B$ other than ENTRY) {
                $\text{IN}[B] = \bigcup_{P \text{ a predecessor of } B} \text{OUT}[P]$;
                $\text{OUT}[B] = gen_B \cup (\text{IN}[B] - kill_B)$;
        }
```

Next iteration

Block B	$\text{OUT}[B]^0$	$\text{IN}[B]^1$	$\text{OUT}[B]^1$	$\text{IN}[B]^2$	$\text{OUT}[B]^2$
B_1	000 0000	000 0000	111 0000	000 0000	111 0000
B_2	000 0000	111 0000	001 1100	111 0111	001 1110
B_3	000 0000	001 1100	000 1110	001 1110	000 1110
B_4	000 0000	001 1110	001 0111	001 1110	001 0111
EXIT	000 0000	001 0111	001 0111	001 0111	001 0111

RD Analysis: An example

Pass 2



entry

B1

d1: i := m-1
d2: j := n
d3: a := u1

GEN[B1]={d1,d2,d3}
KILL[B1]={d4,d5,d6,d7}
IN[B1]=Φ, OUT[B1]={d1,d2,d3}

GEN[B2]={d4,d5}
KILL[B2]={d1,d2,d7}
IN[B2]=Φ
OUT[B2]={d4,d5}

d4: i := i+1
d5: j := j-1

B2

GEN[B2]={d4,d5}
KILL[B2]={d1,d2,d7}
IN[B2]={d1,d2,d3,d7}
OUT[B2]={d3,d4,d5}

d6: a := u2

B3

GEN[B4]={d7}
KILL[B4]={d1,d4}
IN[B4]=Φ
OUT[B4]={d7}
(from pass 1)

d7: i := a+j

B4

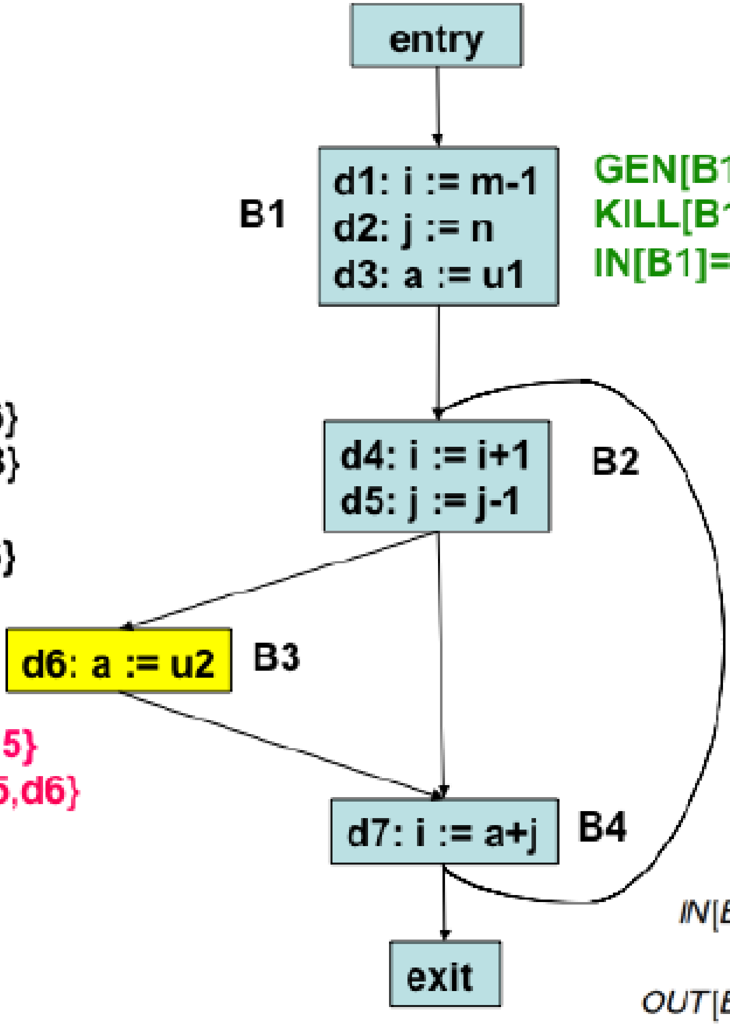
exit

$$IN[B] = \bigcup_{P \text{ is a predecessor of } B} OUT[P]$$

$$OUT[B] = GEN[B] \bigcup (IN[B] - KILL[B])$$

RD Analysis: An example

Pass 2



GEN[B1]={d1,d2,d3}
KILL[B1]={d4,d5,d6,d7}
IN[B1]=Φ, OUT[B1]={d1,d2,d3}

GEN[B2]={d4,d5}
KILL[B2]={d1,d2,d7}
IN[B2]={d1,d2,d3,d7}
OUT[B2]={d3,d4,d5}

GEN[B3]={d6}
KILL[B3]={d3}
IN[B3]=Φ
OUT[B3]={d6}

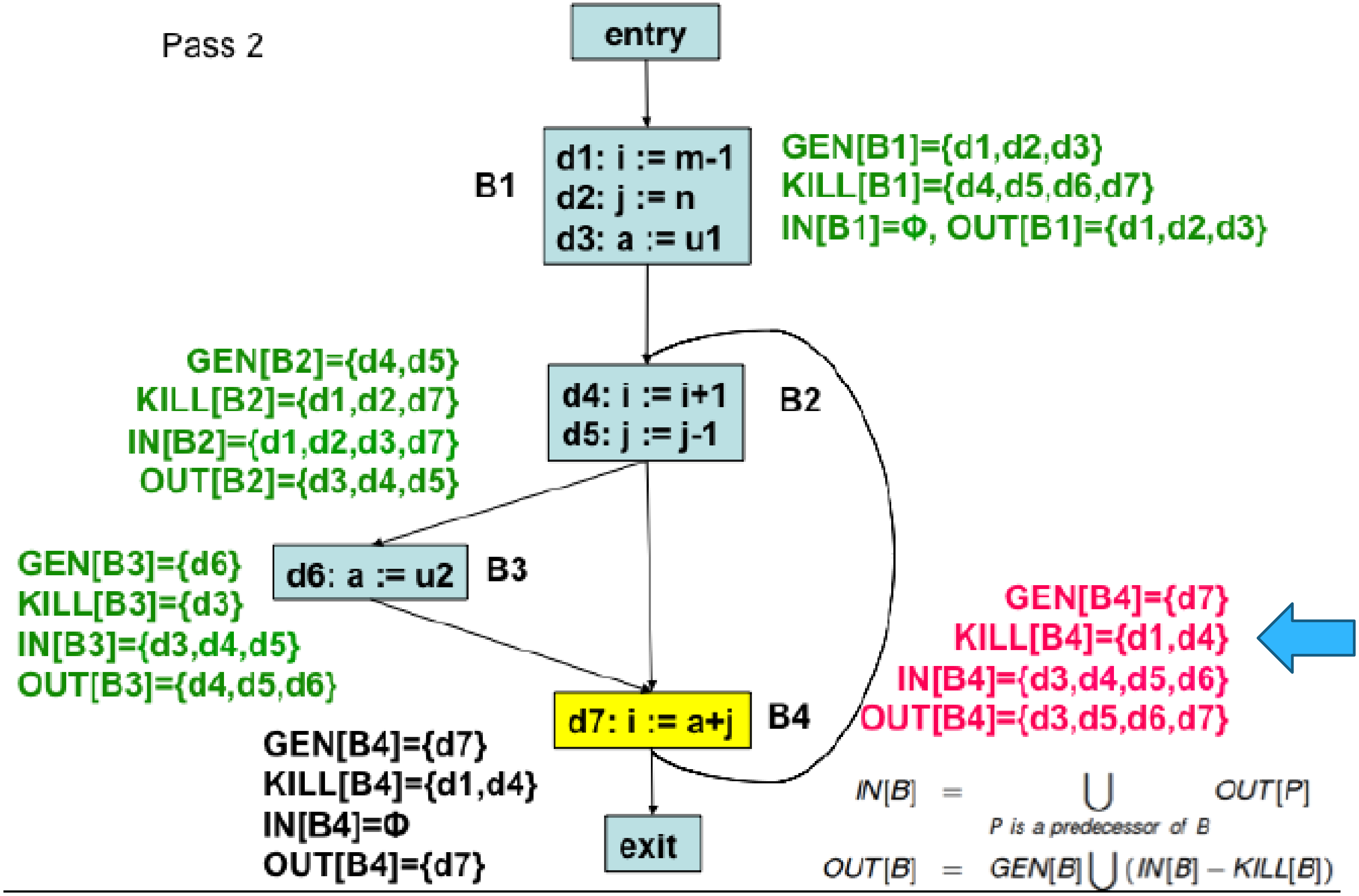
GEN[B3]={d6}
KILL[B3]={d3}
IN[B3]={d3,d4,d5}
OUT[B3]={d4,d5,d6}

$$IN[B] = \bigcup_{P \text{ is a predecessor of } B} OUT[P]$$

$$OUT[B] = GEN[B] \bigcup (IN[B] - KILL[B])$$

RD Analysis: An example

Pass 2



entry

B1

d1: i := m-1
d2: j := n
d3: a := u1

GEN[B1]={d1,d2,d3}
KILL[B1]={d4,d5,d6,d7}
IN[B1]=Φ, OUT[B1]={d1,d2,d3}

B2

d4: i := i+1
d5: j := j-1

GEN[B2]={d4,d5}
KILL[B2]={d1,d2,d7}
IN[B2]={d1,d2,d3,d7}
OUT[B2]={d3,d4,d5}

B3

d6: a := u2

GEN[B3]={d6}
KILL[B3]={d3}
IN[B3]={d3,d4,d5}
OUT[B3]={d4,d5,d6}

B4

d7: i := a+j

GEN[B4]={d7}
KILL[B4]={d1,d4}
IN[B4]=Φ
OUT[B4]={d7}

GEN[B4]={d7}
KILL[B4]={d1,d4}
IN[B4]={d3,d4,d5,d6}
OUT[B4]={d3,d5,d6,d7}

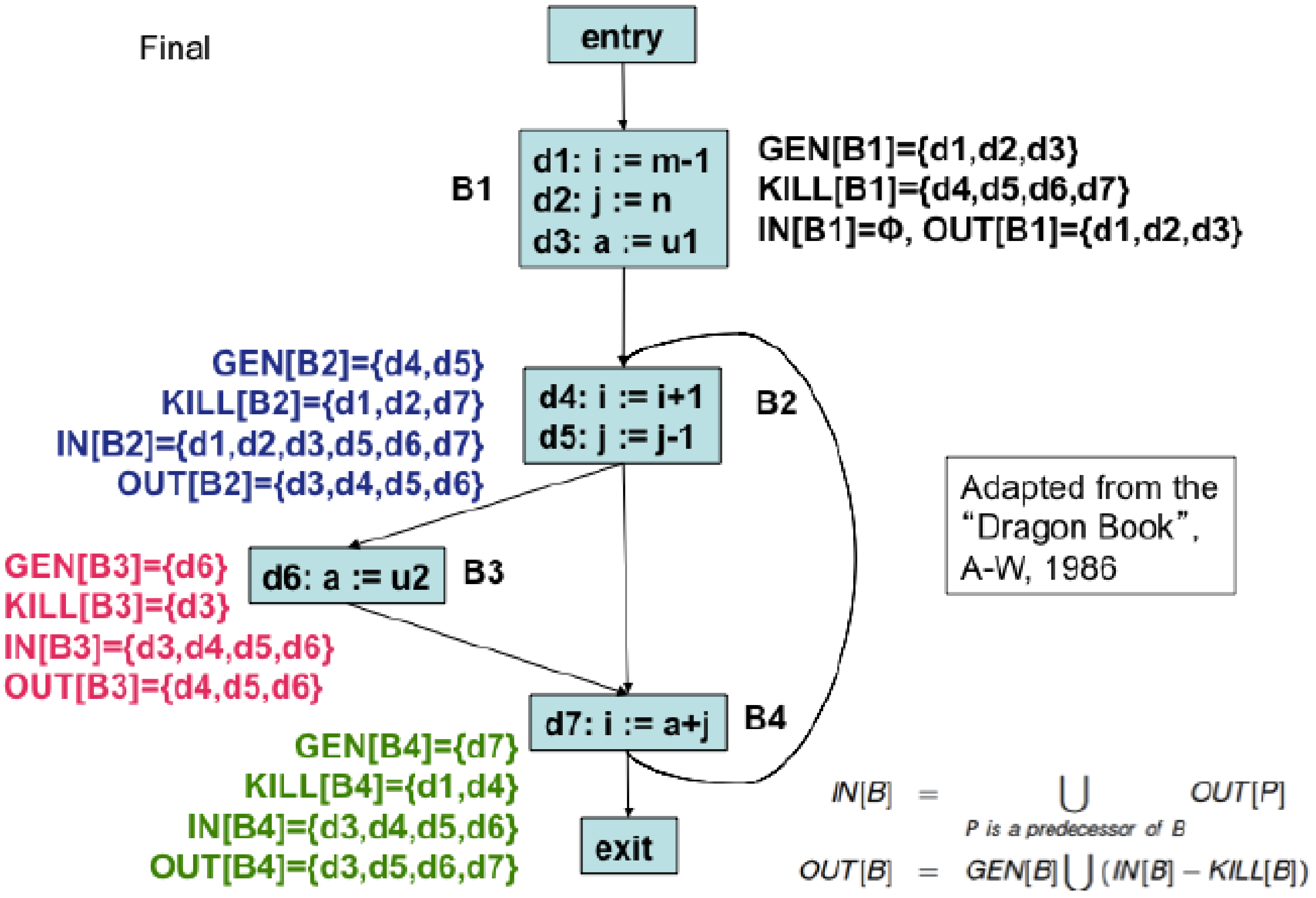
exit

$$IN[B] = \bigcup_{P \text{ is a predecessor of } B} OUT[P]$$

$$OUT[B] = GEN[B] \bigcup (IN[B] - KILL[B])$$

RD Analysis: An example

Final



GEN[B1]={d1,d2,d3}
KILL[B1]={d4,d5,d6,d7}
IN[B1]=Φ, OUT[B1]={d1,d2,d3}

GEN[B2]={d4,d5}
KILL[B2]={d1,d2,d7}
IN[B2]={d1,d2,d3,d5,d6,d7}
OUT[B2]={d3,d4,d5,d6}

GEN[B3]={d6}
KILL[B3]={d3}
IN[B3]={d3,d4,d5,d6}
OUT[B3]={d4,d5,d6}

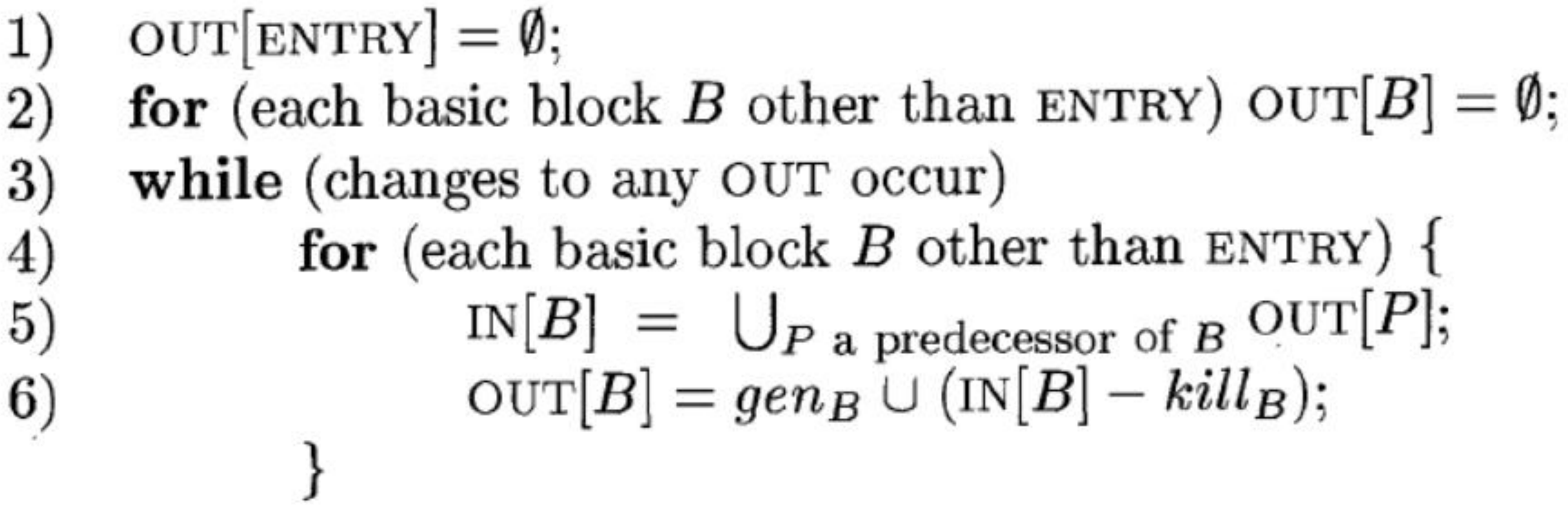
GEN[B4]={d7}
KILL[B4]={d1,d4}
IN[B4]={d3,d4,d5,d6}
OUT[B4]={d3,d5,d6,d7}

Adapted from the "Dragon Book", A-W, 1986

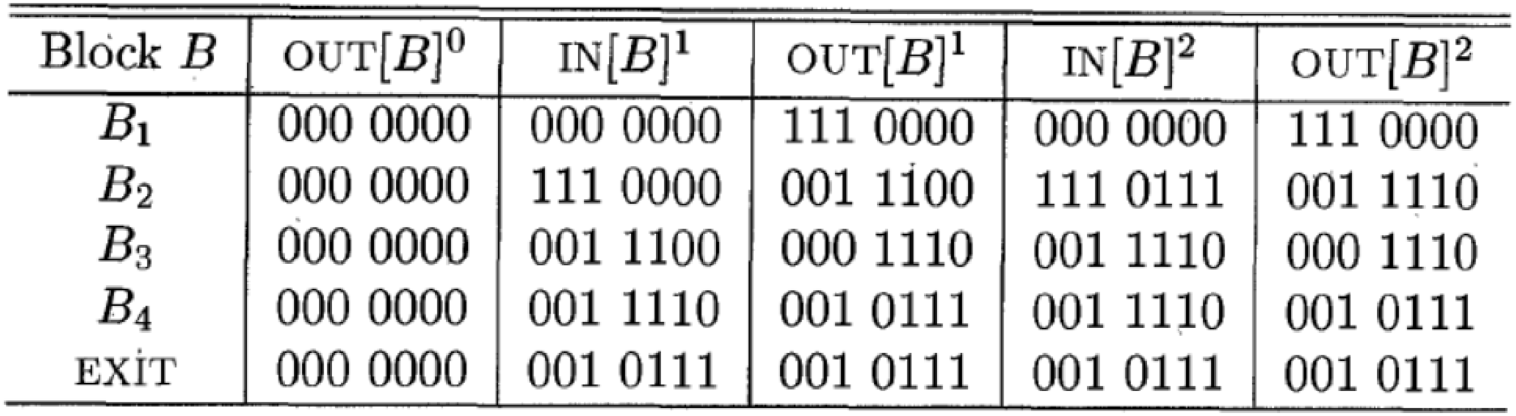
$$IN[B] = \bigcup_{P \text{ is a predecessor of } B} OUT[P]$$

$$OUT[B] = GEN[B] \bigcup (IN[B] - KILL[B])$$

RD algorithm



```
OUT[ENTRY] = $\emptyset$;
for (each basic block $B$ other than ENTRY) OUT[$B$] = $\emptyset$;
while (changes to any OUT occur)
        for (each basic block $B$ other than ENTRY) {
                IN[$B$] = $\bigcup_{P \text{ a predecessor of } B}$ OUT[$P$];
                OUT[$B$] = $gen_B \cup$ (IN[$B$] $-$ $kill_B$);
        }
```



Block B	OUT$[B]^0$	IN$[B]^1$	OUT$[B]^1$	IN$[B]^2$	OUT$[B]^2$
B_1	000 0000	000 0000	111 0000	000 0000	111 0000
B_2	000 0000	111 0000	001 1100	111 0111	001 1110
B_3	000 0000	001 1100	000 1110	001 1110	000 1110
B_4	000 0000	001 1110	001 0111	001 1110	001 0111
EXIT	000 0000	001 0111	001 0111	001 0111	001 0111