DEMANDS: Distributed Energy Management Using Noncooperative Scheduling in Smart Grid

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Abstract—In this paper, the problem of energy scheduling and energy exchange between microgrids and customers is studied as a multi-leader multi-follower noncooperative Stackelberg game. The customers act as leaders, and decide the amount of required energy to be taken in each time slot. On the other hand, the microgrids act as the followers, which need to decide the price per unit energy based on the total requested energy by the connected customers. Using variational inequality, it is shown that the proposed *distributed energy management using scheduling* (DEMANDS) scheme has a Nash equilibrium solution, which is also socially optimal. In the proposed scheme, DEMANDS, each customer gets energy from any of the available microgrids within a coalition neither by paying higher price per unit energy, nor by waiting for the next time slot for service. The proposed scheme, DEMANDS, which enables the microgrids and the customers to reach the equilibrium state, is evaluated theoretically as well as through simulations.

Index Terms—Generalized Stackelberg game, micro-grid, Nash equilibrium, noncooperation, power management, price optimization.

I. INTRODUCTION

TO ACHIEVE improved quality of service, traditional electrical grids are being modernized as *smart grids*. A smart grid [1] is conceptualized as a cyber-physical system equipped with different sustainable models—energy production, energy distribution, and energy usage. A smart grid also integrates several advanced techniques such as advanced metering infrastructure, energy management system, distributed energy system, intelligent electronic devices, and plug-in hybrid electrical vehicles [1]. Unlike the traditional grid, in which electricity is distributed unidirectionally to the customers having a centralized system, in smart grid, customers can also participate in the distribution of energy by announcing the actual amount of required energy using duplex communication infrastructure. In smart grid, the large-scale electrical grid is divided into smaller geographical areas [2], [3]. The energy demand of each

geographical area is fulfilled by single or multiple microgrids [4] having bidirectional electricity exchange facilities with the substation, and the main grid. In the presence of several microgrids [4], it is desired to allow the customer to choose appropriate microgrids to ensure proper distribution of energy with lower price. If insufficient amount of energy is generated by the microgrids, the customers try to optimize their requested amount of energy, while utilizing the stored energy. Additionally, customers seek opportunity to consume energy from the microgrids, if excess energy is generated by the microgrids, in a time slot. Therefore, a distributed energy management system is required to ensure quality of energy service for each microgrid and the overall smart grid infrastructure. One of the important features in a smart grid is the demand-side energy distribution, which gives the opportunity for flexible energy demand according to the requirements of the customers, and the microgrid decides the price per unit energy based on the energy demand.

The microgrids generate energy typically using renewable energy resources [5] such as wind power, solar energy, and hydro power. Therefore, the amount of generated energy is not fixed at different times in a day. If a microgrid has excess amount of energy, it sells that excess amount to the main grid, or other microgrids having demands of energy. As the requested energy by the customers to each microgrid is discrete, the load on each microgrid does not remain the same in any specific time. Moreover, the existing literature on energy distribution in smart grid considered different energy management schemes, using which each customer is connected with a single microgrid. Thus, in on-peak hours, if the customer is willing to pay a higher price, she/he gets the requested energy at that time slot; otherwise, she/he waits for a significant duration of time to get serviced. To overcome this problem, we need proper *distributed energy management using a scheduling* (DEMANDS) approach. Consequently, the customers need to decide the amount of energy to be requested to the microgrid in each time slot. On the other hand, the microgrids need to decide the price per unit energy to ensure maximum profit. In order to do that, the customers and the microgrids need to maximize the respective payoff values of the utility functions.

In this paper, we propose a noncooperative game theoretic algorithm, named DEMANDS, for *distributed energy management using scheduling*. We use a multi-leader multi-follower Stackelberg game theory to decide the strategies of the customers to fulfill their energy demand while expending lower cost. On the other hand, the microgrids choose strategies

to maximize their profit and properly utilize the generated energy. We summarize the contributions of this paper as follows.

1) We propose an algorithm for DEMANDS for real-time energy consumption in the presence of multiple microgrids in a coalition. Each customer or microgrid, decides his/her/its strategy, based on the local information. Thus, the proposed algorithm, DEMANDS, is distributed, which makes it less vulnerable to system failures.

2) The multi-leader multi-follower Stackelberg game is used to evaluate the optimal strategies of the customers using a noncooperative game, and, in the next stage, the optimal strategies of the microgrids are also decided using another similar approach.

3) We present two different algorithms. The first one is executed at the customer-end to determine the amount of energy to be requested. Each microgrid performs the second algorithm in a noncooperative manner, and decides the price per unit energy in a distributed way, based on the total amount of requested energy.

II. RELATED WORK

In the last few years, lot of research work on smart grid emerged [6]–[21]. Some of the existing literature are discussed in this section. Mondal and Misra [9] proposed a decision making process to form coalitions dynamically between microgrids and customers. However, they did not consider how energy requirement can be scheduled in a distributed manner from the customer-side. Bakker et al. [10] recommended a distributed load management scheme with dynamic pricing, and have modeled it as a network congestion game. However, in this scenario, customer has to wait, as the customer does not have the option to switch to another microgrid. Molderink et al. [11] proposed an algorithm by using the energy in the off-peak, and the on-peak hours, with a virtual power plant, for energy management. Sanseverino et al. [12] studied an algorithm for load shifting and storage device management. The authors proposed that during peak hour, heavy loads should be turned off, and vice versa. However, in this scenario, if the customer does not want any delay in receiving the requested energy, she/he has to pay high price for it. In this situation, using the scheme proposed in the present paper, i.e., DEMANDS, the customer can get energy from the other microgrids with lower price per unit energy, and with much lower delay.

Vytelingum et al. [13] proposed an algorithm, in which the customers choose their strategies based on their advance knowledge about the market. They did not consider scheduling in distributed energy management. Fang et al. [22] proposed different energy management schemes. However, in this paper, new opportunities for improved residential energy management and bill reduction are studied without considering the impact of scheduling approach for distributed energy management. Erol–Kantarci and Mouftah [23] studied a time-to-use aware-energy management scheme. In this scheme, a customer consumes energy according to the time. Yet, the energy management policy adopted by the customers and the microgrids need further
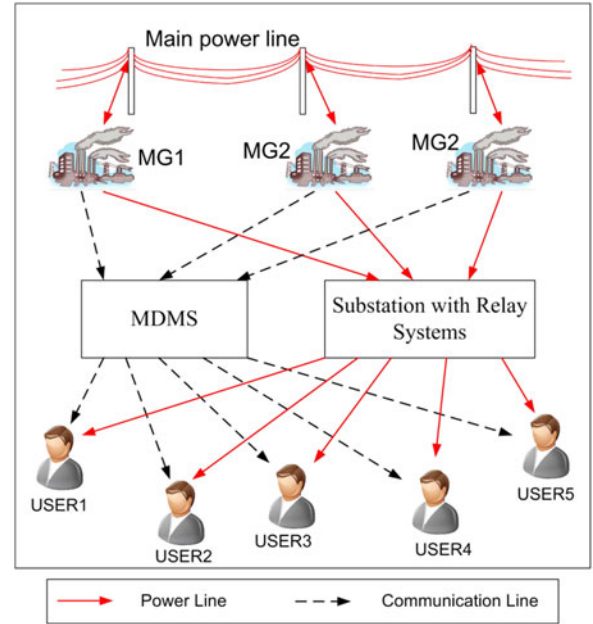

Fig. 1.   Schematic diagram of DEMANDS.

research to have an optimal solution and with minimum delay and less message overhead.

Pipattanasomporn et al. [24] considered smart grid as a multi-agent system, which is visualized to be a combination of several agents working in cooperation to achieve a goal. The authors decompose the complex problem into multiple fragments, namely, control, distributed energy resource, user and database. However, in this paper, we consider an oligopolistic market, where multiple customers and multiple microgrids act noncooperatively, while attempting to maximize his/her/its own payoff, individually.

In contrast to the existing works, a model is used in this paper to characterize the effect of DEMANDS in the smart grid. We use the multi-leader multi-follower Stackelberg game to develop an optimal solution for DEMANDS for the customers, where each customer has multiple option for choosing microgrids.

III. SYSTEM MODEL

We consider a distributed energy management system with multiple microgrids and multiple customers. Each customer connected with multiple microgrids is visualized using relays in the electrical network. Each customer has a dedicated relay to switch connection between the available microgrids, which are available with the coalition. We consider that each customer is not connected with multiple microgrids or main grid, simultaneously. The schematic diagram of the proposed scheme, DEMANDS, is shown in Fig. 1. At each time slot, each customer chooses one suitable microgrid and consumes energy from the selected microgrid among the available microgrids in the coalition. On the other hand, each microgrid does not depend on other microgrids for energy transmission to the customers-end. We consider that, in this system, each microgrid $m \in \mathbb{M}$, where $\mathbb{M}$ is the set of microgrids available in a coalition $\mathbb{C}$, serves the

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electricity demand. Each customer $n \in \mathbb{N}$, where $\mathbb{N}$ is the set of the customers in coalition $\mathbb{C}$, demands x_n^t amount of energy to the microgrid m in time slot $t \in \mathbb{T}$, where $\mathbb{T}$ is the set of the time slots in a day. Therefore, in time slot t, the total energy demanded from the microgrid m by the customers $\mathbb{N}_m \subseteq \mathbb{N}$, where $\mathbb{N}_m$ is the set of the customers who request energy to the microgrid m, is $\mathbb{D}_m^t$. Mathematically

$$\mathbb{D}_m^t = \sum_{n \in \mathbb{N}_m}^{\mathbb{N}_m \subseteq \mathbb{N}} x_n^t \quad \forall t \in \mathbb{T}. \tag{1}$$

The total demand to microgrid m by the customers $\mathbb{N}_m$ must satisfy the following inequality

$$\mathbb{D}_m^t \leq \mathbb{G}_m^t \quad \forall m \in \mathbb{M} \tag{2}$$

where $\mathbb{G}_m^t$ is the total generation capacity of the microgrid m in the time slot t.

Given the amount of energy requested by the customers $\mathbb{N}_m$, each microgrid m sets a price p_m^t to maximize its revenue from supplying energy by strategically choosing the optimal value for its price coefficient.

For completing energy trading successfully, the customers and the microgrids exchange messages with one another, and agree on the energy trading parameters—the amount of required energy by each customer n, x_n^t, and the price per unit energy for microgrid m, p_m^t, that satisfy the objectives of both players, i.e., the customers and the microgrids. We consider that in a day, the amount of required energy by each customer n, e_n, is predicted on a day-ahead basis, based on the prediction of the maximum energy requirement by the appliances installed on the customer-side[25]. However, x_n^t is determined by the tradeoff between the total required energy of each customer n, and p_m^t decided by the microgrid m. We represent the *price vector*, $\vec{\mathbb{P}}^t$, as the collection of price per unit energy, p_m^t, as defined in Definition 1. The demanded energy by each customer n in time slot t, i.e., x_n^t, must satisfy the following constraints:

$$e_n \leq \sum_{t=1}^{t \in \mathbb{T}} x_n^t, \text{ and } \sum_{m=1}^{m \in \mathbb{M}} \sum_{t=1}^{t \in \mathbb{T}} \mathbb{G}_m^t + \Delta_m \geq \sum_{n=1}^{n \in \mathbb{N}} \sum_{t=1}^{t \in \mathbb{T}} x_m^t \tag{3}$$

where Δ_m is the amount of energy consumed by the microgrid m from the main grid. Moreover, the price per unit energy decided by the microgrid m, p_m^t, is dependent on the total amount of demanded energy by the customer $\mathbb{N}_m$, i.e., $\sum_{n \in \mathbb{N}_m} x_n^t$, and the number of customers requested energy at that time slot t, i.e., $|\mathbb{N}_m^t|$. The energy generation by each microgrid depends on the renewable energy resources and external environmental factors. Hence, the amount of generated energy cannot be modified for a fixed time slot. Thereby, the total generation capacity of the microgrids $\mathbb{M}$ within a coalition, i.e., $\sum_{m=1}^{m \in \mathbb{M}} \mathbb{G}_m^t$, is unchangeable for a time slot t, as well as, the individual generation capacity of each microgrid m for a time slot t, $\mathbb{G}_m^t$, is also unchangeable. However, $\sum_{m=1}^{m \in \mathbb{M}} \mathbb{G}_m^t$ and $\mathbb{G}_m^t$ are not constants. Therefore, if a microgrid m needs excess amount of energy, it needs to request the main grid for deficient amount of energy.

Definition 1: For each time slot t, the price vector $\vec{\mathbb{P}}^t$ is defined by the vector with components having information

Type (Request)	CustomerID	Energy (MWh)	SelectionFlag
1 Byte	2 Byte	1 Byte	2 Byte

Fig. 2.   Request message.

Type (Reply)	GridID	Price (USD)	Generated Energy (MWh)
1 Byte	2 Byte	2 Byte	2 Byte

Fig. 3.   Reply message.

about the price per unit energy decided by each microgrid m. Mathematically, we define the price vector $\vec{\mathbb{P}}^t$ as — $\vec{\mathbb{P}}^t = \{p_1^t, p_2^t, \ldots, p_m^t, \ldots, p_{|\mathbb{M}|}^t\}^T, \quad \forall t \in \mathbb{T}$.

Therefore, the energy requested by each customer n has to fulfill the inequalities given in (2) and (3). It also affects the price per unit energy decided by each microgrid m. Thus, the main challenges facing the development of DEMANDS approach are

1) Modeling the decision making processes, and the interaction between the microgrids and the customers.
2) Developing an algorithm, i.e., DEMANDS, for customers such that they can decide the optimum energy to be requested in each time slot t, given the price vector $\vec{\mathbb{P}}^t$ decided by the microgrids $\mathbb{M}$.
3) Each microgrid m decides the price per unit energy, i.e., p_m^t, based on the total demanded energy, $\mathbb{D}_m^t$, while ensuring revenue maximization.

Communication Between Microgrids and Customers

We assume that the communication infrastructure between the customers and the microgrids is based on wireless mesh network. We use the IEEE 802.11b protocol for communication between the microgrids and the customers. First, each customer decides the amount of energy to be requested, and sends a request message. The request message format is shown in Fig. 2. Based on the requests by the customers, each microgrid decides the price per unit energy, and sends a reply message back to the customers. The reply message format is shown in Fig. 3.

IV. DEMANDS: THE PROPOSED NONCOOPERATIVE GAME THEORETIC SOLUTION

A. Justification for the Use of Multi-Leader Multi-Follower Stackelberg Game

In a distributed energy management scenario, each customer tries to consume high amount of energy in order to fulfill his/her energy requirement. However, having option for connecting one of the multiple microgrids available in the coalition, she/he needs to decide the amount of energy to be consumed, while paying less. On the other hand, energy requested by each customer has a cumulative effect on the price decided by the microgrids. Thereby, we use a *multi-leader multi-follower Stackelberg game*, where the customer act as the leaders, and the microgrids act as the followers. This interaction has a similarity with "oligopolistic market," where the request and supply trade-

off is to be maintained in order to maximize individual profit of each player.

B. Game Formulation

To study the interaction between the microgrids and the customers, we use a multi-leader multi-follower Stackelberg game [26]. Multi-leader multi-follower Stackelberg game is a multistage and multilevel game. Here, the customers, who act as leaders, decide the energy to be consumed from which microgrid based on the price vector defined by the microgrids, independently. On the other hand, the microgrids act as the followers, and decide the price per unit energy based on the total demanded energy, independently. In this paper, we follow an extended game formulation approach used by Tushar *et al.* [27]. We define the strategic form, $\boldsymbol{\xi}$, of the proposed noncooperative game as follows:

$$\boldsymbol{\xi} = \langle (\mathbb{N} \cup \mathbb{M}), (e_n, x_n^t, \mathscr{U}_n)_{n \in \mathbb{N}}, (p_m^t, \mathscr{B}_m)_{m \in \mathbb{M}}, (\vec{\mathbb{P}}^t)_{t \in \mathbb{T}} \rangle. \tag{4}$$

The components of strategic form $\boldsymbol{\xi}$ are as follows.

1) Each customer n acts as the leader, and decides x_n^t while satisfying the constraint defined in (3).
2) The utility function $\mathscr{U}_n$ of each customer n captures the benefit of consuming x_n^t amount of energy in each time slot t.
3) The utility function $\mathscr{B}_m$ of each microgrid m captures the revenue gained by supplying the total demanded energy, $\mathbb{D}_m^t$, by the $\mathbb{N}_m$ set of customers.
4) The price per unit energy p_m^t is decided by microgrid m for each time slot t. It is also dependent on the total amount of demanded energy by the $\mathbb{N}_m$ set of customers.
5) Price vector $\vec{\mathbb{P}}^t$ is defined as a vector having the information about the price per unit energy decided by the $\mathbb{M}$ microgrids for time slot t within a coalition.

1) Utility Function of a Customer: For each customer n, we define a utility function, $\mathscr{U}_n(x_n^t, \mathbf{x}_{-n}^t, p_m^t)$, to represent the quantified benefit consuming x_n^t in each time slot t. Here, each customer n tries to maximize his/her individual energy consumption, while paying less money. Hence, each customer maximizes his/her payoff of the utility function $\mathscr{U}_n$. Thus, the properties that each customer n must satisfy, are as follows.

1) The utility function $\mathscr{U}_n$ of each customer n is considered to be a nondecreasing and nonnegative function while satisfying the constraint given in (3).
2) For the marginal value of energy consumption, $\mathscr{U}_n$ is considered to be a nonincreasing function.
3) p_m^t decided by microgrid m affects the utility function of each customer n. With higher p_m^t, amount of demanded energy x_n^t decreases.

Therefore, $\mathscr{U}_n$ of each customer n is as follows:

$$\mathscr{U}_n(x_n^t, \mathbf{x}_{-n}^t, p_m^t) = e_n x_n^t - \frac{1}{2} \gamma_n^t (x_n^t)^2 - p_m^t x_n^t \tag{5}$$

where γ_n^t is the *satisfaction factor* of customer n at time slot t, as defined in Definition 2, and $\gamma_n^t \in (0, 1]$, $x_n^t \in [0, \mathbb{G}_m^t - \sum_{i=1, i \neq n}^{i \in \mathbb{N}_m} x_i^t]$, $x_n^t \in [0, e_n - \sum_{\tau=1}^{t-1} x_n^\tau]$, and $\mathbf{x}_{-n}^t = \{x_1^t, x_2^t, \ldots, x_{n-1}^t, x_{n+1}^t, \ldots, x_{|\mathbb{N}_m|}^t\}$.

Definition 2: At time slot t, the satisfaction factor of a customer n, γ_n^t, is a quantified value which is proportionate with the ratio of the total energy consumed till previous time slot $(t-1)$, i.e., $\sum_{\tau=1}^{t-1} x_n^\tau$, and the total required energy in $\mathbb{T}$ time slots, i.e., $\sum_{\tau=1}^{|\mathbb{T}|} x_n^\tau$ or e_n.

From Definition 2, we conclude that satisfaction factor γ_n^τ of customer n at time slot τ is always the same or is higher than the satisfaction factor of customer n at time slot τ', $\gamma_n^{\tau'}$, where $\tau \geq \tau'$. Mathematically

$$\gamma_n^\tau \geq \gamma_n^{\tau'}, \quad \text{if } \tau \geq \tau' \tag{6}$$

We assume that each customer n does not consume higher amount of energy than his/her requirements.

Lemma 1: The satisfaction factor of each customer n can have the maximum value of 1. Mathematically

$$\arg \max_n \gamma_n^t \leq 1 \quad \forall t \in \mathbb{T} \tag{7}$$

Proof: As we assumed that the required energy of each customer n in a day, i.e., e_n, is fixed, and no customer demands higher amount of energy than his/her total requirement. Therefore

$$\arg \max_{n \in \mathbb{N}} e_n \geq \arg \max_{n \in \mathbb{N}} \sum_{t=1}^{t \in \mathbb{T}} x_n^t$$

$$\Rightarrow \arg \max_{n \in \mathbb{N}} \left[\left(\sum_{\tau=1}^{\tau \in \mathbb{T}} x_n^\tau \right) / (e_n) \right] \leq 1. \tag{8}$$

We know that $\gamma_n^t = \arg \max_{n \in \mathbb{N}} [(\sum_{\tau=1}^{\tau \in \mathbb{T}} x_n^\tau)/(e_n)]$. Hence, it is proved that—$\arg \max_n \gamma_n^t \leq 1, \quad \forall t \in \mathbb{T}$. ∎

2) Utility Function of a Microgrid: For each microgrid m, we define a utility function $\mathscr{B}_m(p_m^t, x_n^t(p_m^t))$ to represent the quantified profit by selling $\sum_{n \in \mathbb{N}_m} x_n^t$ at time slot t. By supplying x_n^t, the microgrid m makes a profit of $p_m^t x_n^t$ amount at time slot t. Each microgrid m aims to maximize its revenue by selling the generated energy, $\mathbb{G}_m^t$. Thus, each microgrid m satisfies following properties.

1) Each microgrid m tries to increase the amount of selling energy, i.e., $\sum_{n \in \mathbb{N}_m} x_n^t$, as it increases the revenue, while satisfying the constraints given in (1) and (2).
2) For marginal revenue of a microgrid m, $\mathscr{B}_m$ is considered to be a nonincreasing function. For reaching this marginal revenue state, each microgrid needs to satisfy the condition – $\mathbb{G}_m^t = \sum_{n \in \mathbb{N}_m} x_n^t, \quad \forall m \in \mathbb{M}$.
3) Each microgrid m tries to sell $\mathbb{G}_m^t$ with higher price to maximize the revenue.

Therefore, the utility function $\mathscr{B}_m$ for each microgrid m is as follows:

$$\mathscr{B}_m(p_m^t, x_n^t(p_m^t)) = p_m^t \sum_{n \in \mathbb{N}_m} x_n^t. \tag{9}$$

In the proposed DEMANDS algorithm, the customers control the price per unit energy indirectly, by choosing a microgrid from the available microgrids $\mathbb{M}$. However, the microgrids decide the price per unit energy, p_m^t, where m, using a dynamic pricing model. We defined the dynamic pricing model as

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follows:

$$
p_m^t = \begin{cases} c_m^t, & \text{if } (p_m^t < c_m^t) \text{ and } \left(\mathbb{G}_m^t \geq \sum_{n \in \mathbb{N}_m} x_n^t\right) \\ K, & \text{if } (p_m^t \geq c_m^t) \text{ and } \left(\mathbb{G}_m^t \geq \sum_{n \in \mathbb{N}_m} x_n^t\right) \\ \infty, & \text{if } \left(\mathbb{G}_m^t < \sum_{n \in \mathbb{N}_m} x_n^t\right) \end{cases}
$$
(10)

where c_m^t is the generation cost per unit energy of microgrid m at time slot t, and $K = \mathscr{A}_m \left(\sum_{n \in \mathbb{N}_m} x_n^t\right) + \mathscr{B}_m \left(\sum_{n \in \mathbb{N}_m} x_n^t\right)^2 + \mathscr{C}_m$, as defined in [28], where $\mathscr{A}_m$, $\mathscr{B}_m$, and $\mathscr{C}_m$ are constants for the microgrid m. Here, each customer n tries to maximize his/her utility function $\mathscr{U}_n$, while satisfying the constraints in (1), and (2). However, the energy demand of each customer n leads to the adoption of noncooperative strategy. Therefore, to reach the generalized Nash equilibrium (GNE), each customer n chooses an amount of energy to be requested, such that

$$
\arg\max_{x_n^t} \mathscr{U}_n (x_n^t, \mathbf{x}_{-n}^t, p_m^t)
$$
$$
\Rightarrow \arg\max_{x_n^t} \left[e_n x_n^t - \frac{1}{2}\gamma_n^t (x_n^t)^2 - p_m^t x_n^t \right].
$$
(11)

However, each microgrid m tries to choose an optimum price per unit energy. Therefore, to reach the GNE, each microgrid m chooses a price per unit energy, p_m^t, such that

$$
\arg\max_{p_m^t} \mathscr{B}_m (p_m^t, x_n^t (p_m^t)) \Rightarrow \arg\max_{p_m^t} \left[\sum_{n \in \mathbb{N}_m} p_m^t x_n^t \right].
$$
(12)

Hence, the solution of the proposed DEMANDS algorithm reaches the Stackelberg equilibrium at which all the leaders, i.e., $\mathbb{N}$ customers, reach their optimal amount of requested energy, given the followers' optimal strategies, i.e., the price per unit energy by $\mathbb{M}$ microgrids at their GNE. We define the GNE states of the players, i.e., the followers and the leaders, as defined in Definition 3.

Definition 3: We define the GNE of proposed strategic form ξ of DEMANDS algorithm using a noncooperative game, if and only if, utility function of each customer n, i.e., leader, $\mathscr{U}_n (x_n^t, \mathbf{x}_{-n}^t, p_m^t)$, and the utility function of each microgrid m, i.e., follower, $\mathscr{B}_m (p_m^t, x_n^t (p_m^t))$ satisfy following inequalities defined in (13)

$$
\mathscr{U}_n (x_n^{t*}, \mathbf{x}_{-n}^{t*}, p_m^{t*}) \geq \mathscr{U}_n (x_n^t, \mathbf{x}_{-n}^{t*}, p_m^{t*})
$$
$$
\mathscr{B}_m (p_m^{t*}, x_n^{t*}(p_m^{t*})) \geq \mathscr{B}_m (p_m^t, x_n^{t*}(p_m^t))
$$
(13)

where n, $\sum_{n \in \mathbb{N}_m} x_n^t \leq \mathbb{G}_m^t$, $x_n^{t*} \in \mathbf{x}^{t*}$, m, $p_m^{t*} \in \mathbf{p}^{t*}$, p_m^{t*} is the price per unit energy at Nash equilibrium decided by the microgrid m for time slot t, and x_n^{t*} is the requested energy at Nash equilibrium decided by the customer n for time slot t.

C. Existence of GNE

In multi-player noncooperative game, the existence of an equilibrium pure strategic solution is not guaranteed always.

Hence, we need to determine the existence of GNE in our proposed multi-leader multi-follower game, i.e., DEMANDS. Due to the fact that variational equality is more socially stable than other GNE, as studied by Tushar *et al.* [27], we try to seek the variational equality for the customers, as discussed in Theorem 1.

Theorem 1: If the set of price decided by microgrid $\mathbb{M}$, i.e., $\vec{\mathbb{P}}^t$, is fixed for a time slot t, there exists a variational equality for utility function $\mathscr{U}_n (x_n^t, \mathbf{x}_{-n}^t, p_m^t)$.

Proof: We define the overall utility function $\tilde{\mathscr{U}}$ of $\mathbb{N}_m$ customers as follows:

$$
\tilde{\mathscr{U}}(\cdot) = \sum_{n=1}^{n \in \mathbb{N}_m} \left[e_n x_n^t - \frac{1}{2}\gamma_n^t (x_n^t)^2 \right] - p_m^t \sum_{n=1}^{n \in \mathbb{N}_m} x_n^t.
$$
(14)

Hence, to reach the GNE state, we need to maximize the overall utility function $\tilde{\mathscr{U}}$. Mathematically

$$
\arg\max \tilde{\mathscr{U}} \left(x_1^t, x_2^t, \ldots, x_n^t, \ldots, x_{|\mathbb{N}_m|}^t; p_m^t \right).
$$
(15)

Applying the Karush–Kuhn–Tucker (KKT) conditions with Lagrange multipliers of each nth customer, we get

$$
\nabla_{x_n^t} \mathscr{U}_n (x_n^t, \mathbf{x}_{-n}^t, p_m^t) - \lambda_n^t \nabla_{x_n^t} \left(\sum_{n \in \mathbb{N}_m} x_n^t - \mathbb{G}_m^t \right) = 0
$$
(16)

$$
\lambda_n^t \nabla_{x_n^t} \left(\sum_{n \in \mathbb{N}_m} x_n^t - \mathbb{G}_m^t \right) = 0
$$
(17)

where $\lambda_n^t \geq 0$, and λ_n^t is the Lagrangian multiplier for the customer n at time slot t.

Hence, we extend (16). Therefore, applying the KKT condition over the overall utility function $\tilde{\mathscr{U}}$, we get

$$
\nabla_{\mathbf{x}^t} \tilde{\mathscr{U}} - \lambda^t \nabla_{\mathbf{x}^t} \left(\sum_{n \in \mathbb{N}_m} x_n^t - \mathbb{G}_m^t \right) = 0
$$
(18)

where $\mathbf{x}^t = \{x_1^t, x_2^t, \ldots, x_n^t, \ldots, x_{|\mathbb{N}_m|}^t\}$, and $\lambda^t = \{\lambda_1^t, \lambda_2^t, \ldots, \lambda_n^t, \ldots, \lambda_{|\mathbb{N}_m|}^t\}$. Therefore, by solving $\nabla_{\mathbf{x}^t} \tilde{\mathscr{U}}$, we get

$$
\mathscr{K} = \nabla_{\mathbf{x}^t} \tilde{\mathscr{U}}
$$
$$
= [e_1 - \gamma_1^t x_1^t - p_m^t; \ldots; e_{|\mathbb{N}_m|} - \gamma_{|\mathbb{N}_m|}^t x_{|\mathbb{N}_m|}^t - p_m^t]^T.
$$
(19)

Now, we find the Jacobi matrix of $\mathscr{K}$ as follows:

$$
\mathbf{J}\mathscr{K} = \begin{bmatrix} -\gamma_1^t & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & -\gamma_{|\mathbb{N}_m|}^t \end{bmatrix}.
$$
(20)

As the Jacobian of $\mathscr{K}$ is a diagonal matrix, and all the elements in the diagonal are negative, we can infer that $\mathbf{x}^t$ has a unique solution, i.e., x_n^t is unique, where $n \in \mathbb{N}_m$. Therefore, there exists a variational equality and GNE solutions.

In the proposed DEMANDS algorithm, we also need to determine the existence of GNE for $\mathbb{M}$ microgrids. Therefore, we try to determine the variational equality for this proposed noncooperative game, as discussed in Theorem 2.

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Theorem 2: For time slot t, if the set of requested energy $\mathbf{x}_n$, is fixed, there exists a variational equality for utility function $\mathscr{B}_m(p_m^t, x_n^t(p_m^t))$, where $\forall n$, and $\forall m$.

Proof: We formulate the overall utility function $\tilde{\mathscr{B}}$ of $\mathbb{M}$ microgrids in a coalition, while satisfying the inequality $-p_m^t \geq c_m^t$, as follows:

$$\tilde{\mathscr{B}}(p_1^t, \ldots, p_{\mathbb{M}}^t; x_1^t, \ldots, x_{|\mathbb{N}|}^t) = \sum_{m=1}^{m \in \mathbb{M}} \sum_{n=1}^{n \in \mathbb{N}_m} p_m^t x_n^t \quad (21)$$

where c_m is generation cost per unit energy for microgrid m. Therefore, we need to maximize the overall utility function $\tilde{\mathscr{B}}$ to reach the variational equilibrium solution. Mathematically

$$\arg \max_{m \in \mathbb{M}} \sum_{m=1}^{m \in \mathbb{M}} \sum_{n=1}^{n \in \mathbb{N}_m} p_m^t x_n^t \quad (22)$$

Taking help of Lagrangian multiplier, we apply KKT condition on the utility function $\mathscr{B}_m(p_m^t, x_n^t(p_m^t))$ of each microgrid m. Therefore,

$$\nabla_{p_m^t} \mathscr{B}_m(p_m^t, x_n^t(p_m^t)) - \varphi_m^t \nabla_{p_m^t}[p_m^t - c_m^t] = 0 \quad (23)$$

$$\varphi_m^t \nabla_{p_m^t}[p_m^t - c_m^t] = 0 \quad (24)$$

where $\varphi_m^t \geq 0$, and φ_m^t is defined as the Lagrangian multiplier for microgrid m at time slot t. If we perform similar transformation for other microgrids also, we get the equation for the overall utility function as follows:

$$\nabla_{\mathbf{p}^t} \tilde{\mathscr{B}} - \varphi^t \nabla_{\mathbf{p}^t}[\mathbf{p}^t - \mathbf{c}^t] = 0 \quad (25)$$

where $\mathbf{p}^t = \{p_1^t, p_2^t, \ldots, p_m^t, \ldots, p_{|\mathbb{M}|}^t\}$ and $\mathbf{c}^t = \{c_1^t, c_2^t, \ldots, c_m^t, \ldots, c_{|\mathbb{M}|}^t\}$. Hence, we get

$$\mathscr{L} = \nabla_{\mathbf{p}^t} \tilde{\mathscr{B}} = \left[x_1^t, x_2^t, \ldots, x_m^t, \ldots, x_{|\mathbb{M}|}^t \right]^T \quad (26)$$

where $[\cdot]^T$ defines a transpose matrix of the solution. Therefore, the Jacobian of matrix $\mathscr{L}$ is a *zero matrix*. As $\mathbf{J}\mathscr{L}$ is a zero matrix, we can conclude that for a fixed set of energy requests by $\mathbb{N}_m$ customers, the microgrid m will have a variational equality point for the price per unit amount of energy, p_m^t, for each time slot t.

Therefore, from Theorems 1 and 2, we conclude that the proposed scheme, DEMANDS, has GNE solution for the requested energy by each customer, and the price per unit amount of energy decided by each microgrid within a coalition.

D. Proposed Algorithm

In this section, we formulate the GNE problem among the customers as a variational inequality problem, and proposed an algorithm that leads to optimally social variational equality solution, which, in turn, leads to GNE solutions. Additionally, the proposed scheme, DEMANDS, is not concerned with which type of sources are there with the microgrids, as the proposed scheme is only concerned about the amount of energy generated by the microgrids.

1) Requested Energy Optimization: For a fixed price decided by the microgrids, each customer n decides the amount of energy, x_n^t, to be requested to the microgrid using Algorithm 1.

Algorithm 1: DEMANDS algorithm for each customer

Input: Each micro-grid m decides the optimum price per unit energy, p_m^{t*}.
Output: Each customer n decides the amount of energy to be requested to micro-grid m.
If $\mathscr{U}_n(\tilde{x}_n^t, x_{-n}^{t*}, p_m^{t*}) \not\geq \mathscr{U}_n(x_n^t, x_{-n}^{t*}, p_m^{t*})$ **then**

   Evaluate λ_n^t solving equation, $\lambda_n^t = e_n - \gamma_n^t x_n^t - p_m^{t*}$;
   $\tilde{x}_n^t$ is the modified amount of requested energy;
   Evaluate $\tilde{x}_n^t$ by solving the following equation,
   $\tilde{x}_n^t = \frac{e_n - p_m^t - \lambda_n^t}{\gamma_n^t}$;
   Send the request message with the $\tilde{x}_n^t$ amount of requested energy;

else

   The amount of energy to be requested is fixed;
   The *Fixed Flag* in the message of the customer n is set;

Algorithm 2: DEMANDS algorithm for each micro-grid

Input: Each customer n decides the amount of energy to be requested, x_n^t, to micro-grid m.
Output: Each micro-grid m decides the optimum price per unit energy, p_m^t.
If $\mathscr{B}_m(\tilde{p}_m^t, x_n^{t*}(\tilde{p}_m^t)) \not\geq \mathscr{B}_m(p_m^t, x_n^{t*}(p_m^t))$ **then**

   Evaluate $\tilde{p}_m^t$ using the following equation:
   $\tilde{p}_m^t = \frac{\sum_{n \in \mathbb{N}_m} e_n - \sum_{n \in \mathbb{N}_m} \gamma_n^t x_n^{t*}}{|\mathbb{N}_m|}$;
   Send the reply message with the $\tilde{p}_m^t$ price per unit energy information;

In such situation, the proposed scheme, DEMANDS, performs well. Applying KKT condition, we get

$$\frac{\partial \mathscr{U}_n}{\partial x_n^t} = 0 \Rightarrow e_n - \gamma_n^t x_n^t - p_m^t - \lambda^t = 0 \quad (27)$$

where $\lambda^t \triangleq \lambda_n^t$, $\forall n$, satisfying the condition in (2). Therefore, the marginal condition is as follows:

$$\mathbb{G}_m^t = \sum_{n \in \mathbb{N}_m} x_n^t. \quad (28)$$

However, $\lambda^t(\mathbb{G}_m^t - \sum_{n \in \mathbb{N}_m} x_n^t) = 0$, and $\lambda^t \geq 0$. Therefore,

$$e_n > \gamma_n^t x_n^t + p_m^t. \quad (29)$$

Now, for overall utility function

$$\sum_{n \in \mathbb{N}_m} e_n > \mathbb{G}_m^t \sum_{n \in \mathbb{N}_m} \gamma_n^t + \mathbb{N}_m p_m^t. \quad (30)$$

Equating the condition for variational equality, we get

$$x_n^{t*} = \frac{e_n - p_m^t - \lambda^t}{\gamma_n^t}. \quad (31)$$

2) Price Optimization: Having analyzed the requested energy of the customers, each microgrid tries to find the optimum price p_m^{t*} using Algorithm 2. Hence, the dynamic energy request is taken into account in the proposed scheme, DEMANDS, while using dynamic pricing mechanism. From (16), we infer that

$$p_m^t \leq e_n - \gamma_n^t x_n^t. \quad (32)$$

Considering that the requested energy by customer n is optimal, i.e., $x_n^t = x_n^{t*}$, we modify (32) as follows:

$$p_m^t \leq e_n - \gamma_n^t x_n^{t*}. \quad (33)$$

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TABLE I
SIMULATION PARAMETERS

Parameter	Value
Simulation area	$20 \times 20 \text{ km}^2$
Number of microgrids	10
Number of Customers	500
Customer's minimum requested energy	65 MWh
Customer's maximum requested energy	110 MWh
Microgrid's minimum generated energy	500 MWh
Microgrid's maximum generated energy	750 MWh
Generation cost per MWh energy	10–20 USD

Fig. 4. Consumed energy.

Therefore, for microgrid m, the maximum price per unit energy, i.e., the decided optimum price, p_m^{t*}, is as follows:

$$p_m^{t*} = e_n - \gamma_n^t x_n^{t*}. \tag{34}$$

Hence, each microgrid m chooses the optimum price per unit energy based on the total requested energy, $\sum_n x_n^t$, by $\mathbb{N}_m$ customers.

V. PERFORMANCE EVALUATION

A. Simulation Settings

We considered randomly generated positions for the microgrids and the customers on a MATLAB-based simulation platform. We considered that the microgrids form a coalition based on their geographical location, as discussed in [9]. In this paper, we have taken randomly generated values for the amount of requested energy for each customer and the amount of generated energy by each microgrid at each time slot, as shown in Table I. Therefore, we claim that the proposed scheme, DEMANDS, is able to handle the randomness in the energy generation by the microgrids. We considered that the total required energy of a customer is fixed in a day. Based on the requested energy by all the customers connected with the microgrid, the microgrid decides the optimum price per unit energy based the variational equilibrium solution.

We have evaluated the performance of the proposed algorithm, DEMANDS, by comparing with the online optimal real-time energy distribution algorithm (OORA)[29], and the optimal real-time pricing algorithm (ORPA) [30] through simulations. In OORA, the authors proposed an algorithm for energy distribution with real-time pricing scheme. However, the OORA algorithm is not based on game theory. On the other hand, Samadi *et al.* [30] proposed a real-time pricing algorithm having an energy consumption controller, which is based on a microeconomics approach. However, they did not consider noon-cooperative scheduling. Thus, we endeavored to improve the performance of energy scheduling using the proposed DEMANDS algorithm. In addition to OORA and ORPA, we also simulated another scheme similar to DEMANDS, but does not use game theory. We named this scheme as distributed energy management without game (DEMwoG).
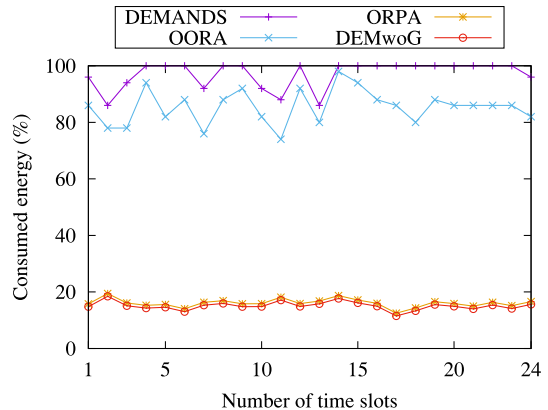
B. Performance Metrics

Requested energy by customers: In a coalition, each customer tries to maximize his/her utility by maximizing the requested energy to the microgrid with lower price.

Excess energy of microgrids: Each microgrid tries to reduce the unused generated energy, i.e., excess energy, by choosing an optimum price per unit energy.

Price per unit energy: The microgrids choose the optimum price per unit energy such that other customers are motivated to request energy, and the microgrid utilizes the generated energy properly in each time slot.

Percentage of energy service served: We evaluate the overall performance with the percentage of customers served. If a customer gets $r\%$ of the initially requested energy, she/he gets $r\%$ of energy service.

C. Results and Discussions

For simulation, we assume that each microgrid monitors the real-time supply and demand in every 5 s interval. After every 5 s, each microgrid has to check the demand-supply curve, and if there is any modification in the demand curve. However, one microgrid may also monitor the supply-demand curve, continuously, which will be computationally expensive. Hence, we chose 5 s interval to reduce the computational complexity, and took advantage of having energy generation and demand side management in smart grid.

In Fig. 4, the percentage of total consumed energy in each time slot within a coalition is shown. Fig. 4 shows that the energy consumed in each time slot is within 90%–100% for DEMANDS, whereas the consumed energy is within 75%–90% for OORA, and 15%–20% for ORPA. Therefore, we can infer that the satisfaction level of the customers is almost 9.6%, and 80% higher using DEMANDS than using OORA, and ORPA, respectively. Fig. 4 shows that the excess amount of energy is much less in DEMANDS than OORA, ORPA, and DEMwoG. From Fig. 4, we conclude that the underutilization of generated energy is 16.67%, 76.75%, 77% higher than DEMANDS in OORA, ORPA, and DEMwoG, respectively. Fig. 5 reestablishes that the utilization of generated energy is much higher using DEMANDS than using OORA, ORPA, and DEMwoG.
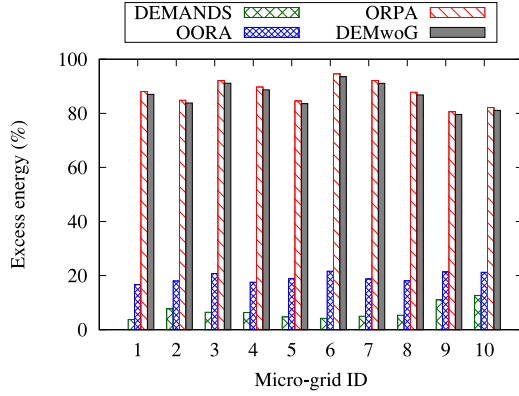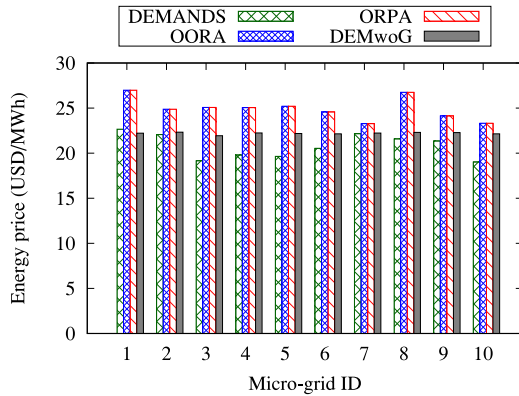
Fig. 5.   Excessed energy.

Fig. 7.   Cumulative profit.

Fig. 6.   Price per unit energy.

Fig. 8.   Customers served over a day.

Fig. 9.   Customer served per micro-grid.

The excess amount of generated energy is 50%–66% higher than DEMANDS, for OORA, ORPA, and DEMwoG. Fig. 6 shows that the price per unit energy is much lower for DEMANDS than OORA, ORPA, and DEMwoG. Therefore, the customers get the required energy with much lower price. Additionally, we consider that the microgrids sell the excess amount of generated energy to the main grid. We consider that the price per unit energy paid by the main grid is constant, i.e., the minimum selling price to the customers, for different schemes, such as DEMANDS, OORA, ORPA, or DEMwoG. Hence, we get that using DEMANDS, the profit of each microgrid is 6.76%, 19.03%, and 18.24% higher than using OORA, ORPA and DEMwoG, respectively, as shown in Fig. 7. Fig. 8 shows that using the proposed DEMANDS scheme, almost 100% request of each customer is served. However, the percentage of customers served in each time slot is within 70%–95%, and 15%–25% using OORA, and ORPA, respectively. Fig. 9 shows that, on an average the percentage of customers served by each microgrid in a day is also higher in DEMANDS than OORA, ORPA, and DEMwoG. In Fig. 9, the percentage of customers served by each microgrid remains almost invariant. However, the percentage of customers served by each microgrid is much lower using ORPA than using DEMANDS. Therefore, we conclude that the energy requested by the customers are more distributed among the available microgrids using dynamic price mod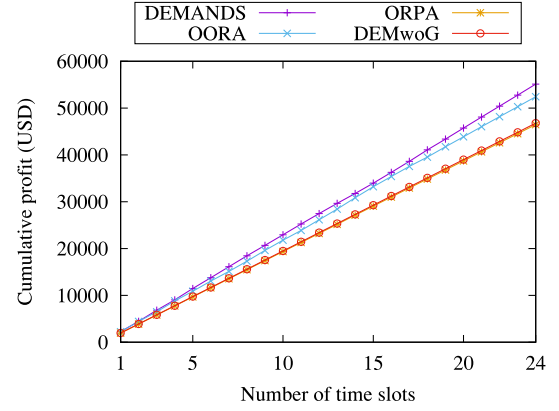el, and the generated energy is uniformly utilized using 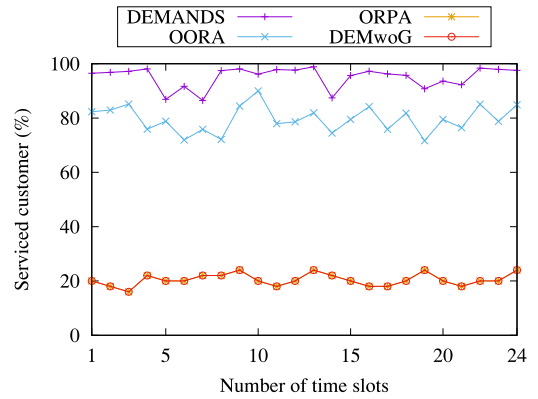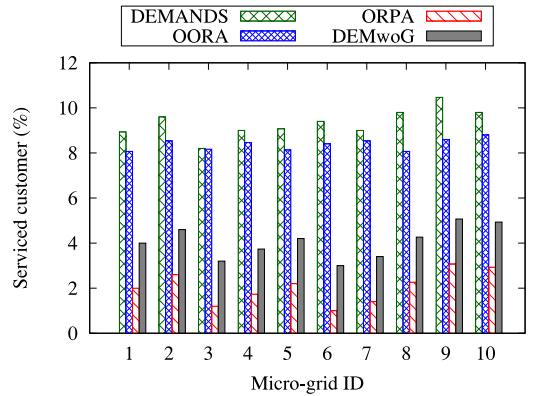DEMANDS than using OORA, ORPA, and DEMwoG. Hence, in addition to ensuring maximum use of renewable energy sources and the minimal use of tradition energy resources, the proposed scheme, DEMANDS, also ensures proper load balancing among the available microgrids.

VI. Conclusion

In this paper, we formulated the multi-leader multi-follower Stackelberg game to study the problem of DEMANDS. Based

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on the proposed algorithm, DEMANDS, we showed how a customer decides the optimum amount of energy to be requested, when each customer is connected with multiple microgrids. The microgrids also choose an optimum price per unit energy, so as to maximize its profit, and utilize its generated energy properly. The simulation results show that the proposed approach yields improved results.

Future extension of this paper includes understanding how the generated energy can be distributed using a centralized management unit. This paper can also be extended by introducing plug-in hybrid electric vehicles in a smart grid. Additionally, this paper can be extended by considering the uncertainty in energy generation by the microgrids and the uncertainty in customers' energy consumption.

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