Multi-Dimensional Arrays

Two Dimensional Arrays

We have seen that an array variable can store a list of values.

Many applications require us to store a table of values.

	Subject 1	Subject 2	Subject 3	Subject 4	Subject 5
Student 1	75	82	90	65	76
Student 2	68	75	80	70	72
Student 3	88	74	85	76	80
Student 4	50	65	68	40	70

Two Dimensional Arrays

	Subject 1	Subject 2	Subject 3	Subject 4	Subject 5
Student 1	75	82	90	65	76
Student 2	68	75	80	70	72
Student 3	88	74	85	76	80
Student 4	50	65	68	40	70

The table contains a total of 20 values, five in each line.

- The table can be regarded as a matrix consisting of four rows and five columns.

C allows us to define such tables of items by using two-dimensional arrays.

Declaring 2-D Arrays

General form:

```
type array_name[row_size][column_size];
```

Examples:

```
int  marks[4][5];
float  sales[12][25];
double  matrix[100][100];
```

First index indicates row, second index indicates column.

Both row index and column index start from 0 (similar to what we had for 1-d arrays)

Declaring 2-D Arrays

```
int  m[4][5];
```

	Column 0	Column 1	Column 2	Column 3	Column 4
Row 0	m[0][0]	m[0][1]	m[0][2]	m[0][3]	m[0][4]
Row 1	m[1][0]	m[1][1]	m[1][2]	m[1][3]	m[1][4]
Row 2	m[2][0]	m[2][1]	m[2][2]	m[2][3]	m[2][4]
Row 3	m[3][0]	m[3][1]	m[3][2]	m[3][3]	m[3][4]

Accessing Elements of a 2-D Array

Similar to that for 1-D array, but use two indices.

- **First index indicates row, second index indicates column.**
- **Both the indices should be expressions which evaluate to integer values.**

Examples:

```
x[m][n] = 0;
c[i][k] += a[i][j] * b[j][k];
val = sqrt( arr[j*3][k+1] );
```

How is a 2-D array stored in memory?

Starting from a given memory location (starting address of the array), the elements are stored row-wise in consecutive memory locations.

- **x: starting address of the array in memory**
- **c: number of columns**
- **k: number of bytes allocated per array element, e.g., sizeof(int)**

- `a[i][j]` **is allocated memory location at address** `x + (i * c + j) * k`

a[0][0] a[0][1] a[0][2] a[0][3] | a[1][0] a[1][1] a[1][2] a[1][3] | a[2][0] a[2][1] a[2][2] a[2][3]

Row 0 | **Row 1** | **Row 2**

Array Addresses

```
int main()
 {
  int a[3][5];
  int i, j;

  for (i=0; i<3;i++)
  {
    for (j=0; j<5; j++)
       printf ("%u\n", &a[i][j]);
    printf ("\n");
  }
  return 0;
 }
```

Output

```
3221224480
3221224484
3221224488
3221224492
3221224496

3221224500
3221224504
3221224508
3221224512
3221224516

3221224520
3221224524
3221224528
3221224532
3221224536
```

How to read the elements of a 2-D array?

By reading them one element at a time

```
for (i=0; i<nrow; i++)
    for (j=0; j<ncol; j++)
        scanf  ("%f", &a[i][j]);
```

- The ampersand (&) is necessary.
- The elements can be entered all in one line or in different lines.

We can also initialize a 2-D array at the time of declaration:

```
int a[MAX_ROWS][MAX_COLS] = { {1,2,3}, {4,5,6}, {7,8,9} };
```

How to print the elements of a 2-D array?

By printing them one element at a time.

```
for (i=0; i<nrow; i++)
    for (j=0; j<ncol; j++)
        printf  ("%f  ", a[i][j]);
```

This will print all elements in one line.

```
for (i=0; i<nrow; i++) {
    for (j=0; j<ncol; j++)
        printf ("%f  ", a[i][j]);
          printf("\n");
}
```

This will print the elements with one row in each line (matrix form).

Example: Matrix addition

```
int main()
{
    int  a[100][100], b[100][100],
                   c[100][100], p, q, m, n;

    printf ("Enter dimensions: ");
    scanf ("%d %d", &m, &n);

    for  (p=0; p<m; p++)
        for  (q=0; q<n; q++)
           scanf ("%d", &a[p][q]);

    for  (p=0; p<m; p++)
        for  (q=0; q<n; q++)
           scanf ("%d", &b[p][q]);
```

```
   for  (p=0; p<m; p++)
      for  (q=0; q<n; q++)
          c[p][q] = a[p][q] + b[p][q];

     for (p=0; p<m; p++)
     {
           for (q=0; q<n; q++)
               printf ("%d ", c[p][q]);
               printf  ("\n");
      }
     return 0;
}
```

A 2-D array is an array or 1-D arrays, and so a row pointer

```
#include <stdio.h>
int main ()
{
   int i, j, A[4][5] = { { 7, 14,  3, 16,  6}, {11,  5,  9, 13, 18},
                         { 2, 15, 20,  1, 19}, {10,  4, 12, 17,  8} };
   for (i=0; i<4; ++i) {
      for (j=0; j<5; ++j) printf("%p ", &A[i][j]);
         printf("\n");
   }
   printf("sizeof(A)  = %3lu,   A = %p,   A + 1 = %p\n", sizeof(A),   A,  A + 1);
   printf("sizeof(*A) = %3lu,  *A = %p,  *A + 1 = %p\n", sizeof(*A), *A, *A + 1);
   printf("sizeof(&A) = %3lu,  &A = %p,  &A + 1 = %p\n", sizeof(&A), &A, &A + 1);
   return 0;
}
```

Output

```
0x7ffc314fe100 0x7ffc314fe104 0x7ffc314fe108 0x7ffc314fe10c 0x7ffc314fe110
0x7ffc314fe114 0x7ffc314fe118 0x7ffc314fe11c 0x7ffc314fe120 0x7ffc314fe124
0x7ffc314fe128 0x7ffc314fe12c 0x7ffc314fe130 0x7ffc314fe134 0x7ffc314fe138
0x7ffc314fe13c 0x7ffc314fe140 0x7ffc314fe144 0x7ffc314fe148 0x7ffc314fe14c
sizeof(A)  =  80,   A = 0x7ffc314fe100,   A + 1 = 0x7ffc314fe114
sizeof(*A) =  20,  *A = 0x7ffc314fe100,  *A + 1 = 0x7ffc314fe104
sizeof(&A) =   8,  &A = 0x7ffc314fe100,  &A + 1 = 0x7ffc314fe150
```

Passing 2-d arrays to functions

Passing 2-D arrays to functions

Similar to that for 1-D arrays.

- The array contents are **not** copied into the function.
- Rather, the address of the first element is passed.

For calculating the address of an element in a 2-D array, the function needs:

- The starting address of the array in memory (say, **x**)
- Number of bytes per element (say, **k**)
- Number of columns in the array, i.e., the size of each row (say, **c**)

> `a[i][j]` is located at memory address x + (i * c + j) * k

The above three pieces of information must be known to the function.

Example

```
int main()
{
    int  a[15][25],  b[15]25];
    …
    …
    add (a, b, 15, 25);
    …
    …
}
```

```
void  add (int x[][25], int y[][25],
                   int rows, int cols)
{

}
```

We can also write

int x[15][25], y[15][25];

The first dimension is ignored. But the second dimension must be given.

Example: Matrix addition with functions

```
void ReadMatrix (int A[][100], int x, int y)
{
    int i, j;
    for (i=0; i<x; i++)
        for (j=0; j<y; j++)
            scanf (“%d”, &A[i][j]);
}
```

```
void AddMatrix( int A[][100], int B[][100], int C[][100], int x, int y)
{
    int i, j;
    for (i=0; i<x; i++)
        for (j=0; j<y; j++)
            C[i][j] = A[i][j] + B[i][j];
}
```

Example: Matrix addition

```
void PrintMatrix (int A[][100], int x, int y)
{
    int i, j;
    printf ("\n");
    for (i=0; i<x; i++)
    {
       for (j=0; j<y; j++)
           printf (" %5d", A[i][j]);
       printf("\n");
    }
}
```

```
int main()
{
    int  a[100][100], b[100][100],
         c[100][100], p, q, m, n;

    scanf ("%d%d", &m, &n);

    ReadMatrix(a, m, n);
    ReadMatrix(b, m, n);

    AddMatrix(a, b, c, m, n);

    PrintMatrix(c, m, n);
    return 0;
}
```

Example:

```
#include  <stdio.h>
int main() {
  int  a[15][25], b[15][25], c[15][25];
  int m, n;
  scanf ("%d %d", &m, &n);
  for (p=0; p<m; p++)
     for  (q=0; q<n; q++)
        scanf ("%d", &a[p][q]);
  for (p=0; p<m; p++)
     for (q=0; q<n; q++)
        scanf ("%d", &b[p][q]);
  add (a, b, m, n, c);
  for (p=0; p<m; p++)  {
     for  (q=0; q<n; q++)
        printf("%f   ", c[p][q]);
     printf("\n");
    }
}
```

```
void add (int x[][25], int y[][25], int m,
          int n, int z[][25])
{
     int p, q;
     for (p=0; p<m; p++)
       for (q=0; q<n; q++)
          z[p]q] = x[p][q] + y[p][q];
}
```

Note that the number of columns has to be fixed in the function definition.

- There is no difference between
  void add(int x[ ][25], …) and
  void add(int x[15][25], …)
- Specifying the first dimension is not necessary, but not a mistake.

Example: Transpose of a matrix

```
#include <stdio.h>

void transpose (int x[][3], int n)
{
   int  p, q, t;

   for (p=0; p<n; p++)
     for (q=0; q<n; q++)
      {
         t = x[p][q];
         x[p][q] = x[q][p];
         x[q][p] = t;
       }
}
```

```
main()
{
   int a[3][3], p, q;

   for (p=0; p<3; p++)
     for (q=0; q<3; q++)
       scanf ("%d", &a[p][q]);

   transpose (a, 3);

   for (p=0; p<3; p++)
   {
     for (q=0; q<3; q++)
        printf ("%d  ", a[p][q]);
     printf ("\n");
   }
}
```

Example: Transpose of a matrix

```
#include <stdio.h>

void transpose (int x[][3], int n)
{
    int  p, q, t;

    for (p=0; p<n; p++)
      for (q=0; q<n; q++)
       {
           t = x[p][q];
           x[p][q] = x[q][p];
           x[q][p] = t;
        }
}
```

```
main()
{
   int a[3][3], p, q;

   for (p=0; p<3; p++)
     for (q=0; q<3; q++)
       scanf ("%d", &a[p][q]);

   transpose (a, 3);

   for (p=0; p<3; p++)
   {
     for (q=0; q<3; q++)
        printf ("%d  ", a[p][q]);
     printf ("\n");
   }
}
```

This function is wrong. Why?

The Correct Version

```
void transpose (int x[][3], int n)
{
   int  p, q, t;

   for (p = 0; p < n; p++)
     for  (q = p; q < n; q++)
       {
         t = x[p][q];
         x[p][q] = x[q][p];
         x[q][p] = t;
       }
}
```

10	20	30
40	50	60
70	80	90

10	40	70
20	50	80
30	60	90

Dynamically allocating 2-d arrays
A brief discussion

You may recall …

We have discussed the issue of dynamically allocating space for 1-D arrays

- Using `malloc()` library function.

```
int *ptr;

ptr = (int*) malloc( 100 * sizeof(int) );
```

How to dynamically allocate a 2-d array?

Many variations possible:

1. **Fixed number of rows, but variable number of columns**
2. **Variable number of rows, but fixed number of columns**
3. **Both number of rows and columns variable**

We will discuss only the first variation:

Fixed number of rows, but variable number of columns

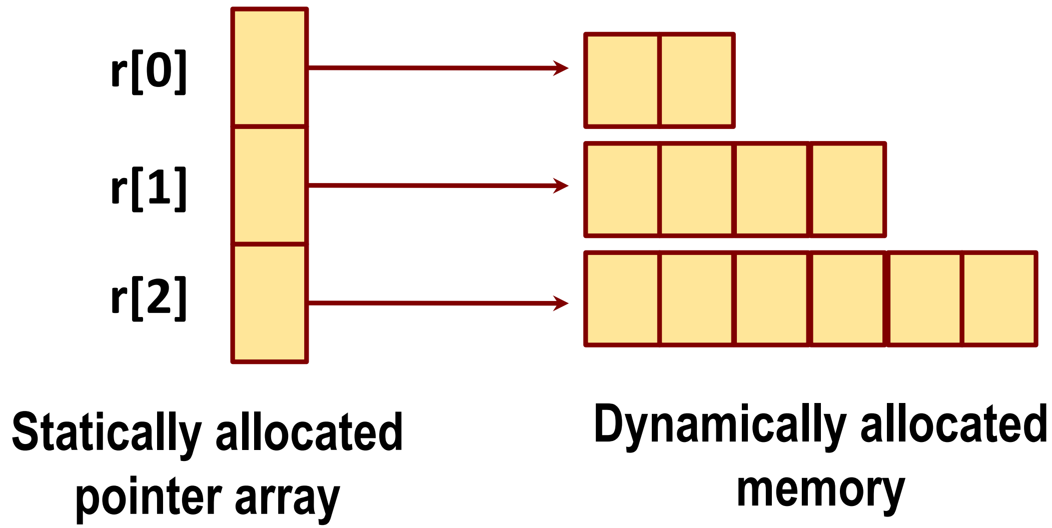
Fixed number of rows, but variable number of columns

Let us assume the number of rows is fixed to 3.

We can use an array of pointers of size 3, where the i^th element of this array (a pointer) will point to the i^th row of the 2-d array.

```
int  *r[3], i, c;
printf ("Enter nos. of columns of the 2-d array:");
scanf("%d", &c);                          // each row will have c elements
for (i=0;i<3;i++)
  r[i] = (int *) malloc(c*sizeof(int)); // allocate i-th row
```

Possible to have rows with different number of elements



```c
#include <stdio.h>
#include <stdlib.h>
int main()
{
  int *r[3], i, j, col;
  for (i=0; i<3; ++i) {
    col = 2 * (i+1);
    r[i] = (int *) malloc (col*sizeof(int));
    for (j=0; j<col; ++j)
      r[i][j] = i + j;
  }
 for (i=0; i<3; ++i) {
    col = 2 * (i+1);
    for (j=0; j<col; ++j)
       printf("%d ", r[i][j]);
    printf("\n");
  }
  return 0;
}
```

Output

```
0 1
1 2 3 4
2 3 4 5 6 7
```

We have studied only 2-d arrays.
C allows arrays of higher dimensions as well.

Practice problems

1. Write a function that takes an n x n square matrix A as parameter (n < 100) and returns 1 if A is an upper-triangular matrix, 0 otherwise.
2. Repeat 1 to check for lower-triangular matrix, diagonal matrix, identity matrix.
3. Consider a n x n matrix containing only 0 or 1. Write a function that takes such a matrix and returns 1 if the number of 1's in each row are the same and the number of 1's in each column are the same; it returns 0 otherwise.
4. Write a function that reads in an m x n matrix A and an n x p matrix B, and returns the product of A and B in another matrix C. Pass appropriate parameters.
5. Write a function to find the transpose of a non-square matrix A in a matrix B.
6. Repeat the last exercise when the transpose of A is computed in A itself. Use no additional 2-d arrays.

For each of the above, also write a main function that reads the matrices, calls the function, and prints the results (a message, the result matrix etc.)

ADVANCED TOPICS

Pointers equivalent to two-dimensional arrays

Generalization from one-dimensional arrays

Consider the statically allocated 1-d array:

```
int A[20];
```

A pointer that can browse through A is declared as:

```
int *p;
```

Such a pointer can be allocated dynamic memory and freed as:

```
p = (int *)malloc(20 * sizeof(int));
free(p);
```

- **What are the analogous pointers for 2-d arrays that you have seen earlier?**
- **How can these pointers be allocated and deallocated their own memory?**

What are our 2-d arrays?

We have seen two types of 2-d arrays:

```
int A[10][20];
int *B[10];
```

Both these arrays are statically allocated.

- A is an array of arrays, and has no dynamic component.
- B is an array of pointers. Individual pointers in B[] can be dynamically allocated.

As statically allocated arrays, both A and B suffer from the two standard disadvantages:

- Waste of space
- Inadequacy to handle larger than the allocated space

Dynamic versions of A and B overcome these shortcomings.

Dynamic version of A

```
int A[10][20];
```

A pointer matching A should be a pointer to an array of 20 int variables.

But

```
int *p[20];
```

declares an array of 20 int pointers, not a pointer to an array.

Three ways of defining the correct pointer equivalent to A:

Method 1:
```
int (*p)[20];
```

Method 2:
```
typedef int row[20];
row *p;
```

Method 3:
```
typeof(int [20]) *p;
```
// Not available in the original C specification

In all the cases, p is a *single* pointer.

Dynamic version of B

```
int B[10];
```

B is an array of 10 int pointers.
The equivalent pointer is a pointer to an int pointer.

```
int **q;
```

A 2-d array declared by q is fully dynamic.

- The number of rows can be decided during the run of the program.
- The size of each row can also be decided *individually* during the run.

Note: It is *illegal* to set `q = A;` or `p = B;` Expect segmentation fault if you do so (ignoring the warnings issued by the compiler).

Dynamic memory for p

p is a single pointer, and can be allocated and deallocated memory in a single shot.

- **Method 1:**
  ```
  p = (int (*)[20])malloc(10 * 20 * sizeof(int));
  ```
- **Method 2:**
  ```
  p = (row *)malloc(10 * sizeof(row));
  ```
- **Method 3:**
  ```
  p = (typeof(int [20]) *)malloc(10 * 20 * sizeof(typeof(int [20])));
  ```

Freeing requires only one call.

```
free(p);
```

Dynamic memory for q

First, you allocate the required number of row headers, and then the rows individually.

```
q = (int **)malloc(10 * sizeof(int *));
for (i=0; i<10; ++i)
   q[i] = (int *)malloc(20 * sizeof(int));
```

Freeing is also a multi-step process.

```
for (i=0; i<10; ++i) free(q[i]);
free(q);
```

Note: Free the individual rows *before* freeing the array of row headers.

Example: Vandermonde matrices

A Vandermonde matrix corresponding to *n* real-valued elements $a_0, a_1, \ldots, a_{n-1}$ is defined as:

1	1	1	…	1
a_0	a_1	a_2	…	a_{n-1}
a_0^2	a_1^2	a_2^2	…	a_{n-1}^2
⋮	⋮	⋮	…	⋮
a_0^{n-1}	a_1^{n-1}	a_2^{n-1}	…	a_{n-1}^{n-1}

An application works with Vandermonde matrices for $n \le 100$. A static 2-d array would require a total storage of 100 x 100 = 10,000 cells. This leads to waste if *n* is small.

We write a function genvdm(A,n) that obtains $a_0, a_1, \ldots, a_{n-1}$ from the 1-d array A, and returns a pointer to an dynamically allocated array of rows.

The row size must be fixed beforehand. But we can allocate exactly *n* rows to reduce wastage.

Dynamic memory for storing Vandermonde matrices

```
#include <stdio.h>
#include <stdlib.h>

#define MAXDIM 100

double (*genvdm ( double *A, int n )) [MAXDIM]
{
   double (*p)[MAXDIM];
   int i, j;
   p = (double (*)[MAXDIM])malloc(n * MAXDIM * sizeof(double));
   for (i=0; i<n; ++i) {
      // i is an index in A, and a column in p. j is a row in p.
      p[0][i] = 1;
      for (j=1; j<n; ++j) p[j][i] = p[j-1][i] * A[i];
   }
   return p;
}

void prnvdm ( double M[][MAXDIM], int n )
{
   int i, j;
   for (i=0; i<n; ++i) {
      for (j=0; j<n; ++j) printf("%10.5lf ", M[i][j]);
      printf("\n");
   }
}
```

Storage of Vandermonde matrices (continued)

```
int main ()
{
   double A[MAXDIM], (*V)[MAXDIM];
   int n, i;

   printf("Enter dimension of V: "); scanf("%d", &n);
   printf("Enter %d elements: ", n);
   for (i=0; i<n; ++i) scanf("%lf", A+i);
   V = genvdm(A,n);
   prnvdm(V,n);
   exit(0);
}
```

Output

```
Enter dimension of V: 5
Enter 5 elements: -1 0.1 1.1 2.5 3.2
   1.00000    1.00000    1.00000    1.00000      1.00000
  -1.00000    0.10000    1.10000    2.50000      3.20000
   1.00000    0.01000    1.21000    6.25000   10.24000
  -1.00000    0.00100    1.33100   15.62500   32.76800
   1.00000    0.00010    1.46410   39.06250  104.85760
```

Exercise: Free the 2-d memory allocated to V.

Antisymmetric matrices

A symmetric matrix is an n x n matrix with $a_{ij} = a_{ji}$ for all i, j.

An antisymmetric matrix is an n x n matrix with $a_{ij} = -a_{ji}$ for all i, j. Since $a_{ii} = -a_{ii}$, the major diagonal is filled by 0. Moreover, the entries below the main diagonal can be obtained from the entries above the main diagonal.

0	5	3	–2	4
–5	0	–6	–1	0
–3	6	0	2	7
2	1	–2	0	1
–4	0	–7	–1	0

We use a fully dynamic 2-d array to store only the elements above the main diagonal.

The function genasm(n) returns a pointer to this array given n as input. Here, we take $a_{ij} = i - j$.

Compact storage of an antisymmetric matrix

```
#include <stdio.h>
#include <stdlib.h>

int **genasm ( int n )
{
   int **q, i, j;

   q = (int **)malloc((n-1) * sizeof(int *));
   for (i=0; i<n-1; ++i) {
      q[i] = (int *)malloc((n-i-1) * sizeof(int));
      for (j=i+1; j<n; ++j) q[i][j-i-1] = i-j;
   }
   return q;
}
```

Storage of antisymmetric matrices (continued)

```
void prnasm ( int *U[], int n )
{
   int i, j;
   for (i=0; i<n; ++i) {
      for (j=0; j<i; ++j) printf("%3d ", -U[j][i-j-1]);
      printf("  0 ");
      for (j=i+1; j<n; ++j) printf("%3d ", U[i][j-i-1]);
      printf("\n");
   }
}
int main ()
{
   int **U, n;
   printf("Enter dimension (n): "); scanf("%d", &n);
   U = genasm(n);
   prnasm(U,n);
   exit(0);
}
```

Exercise: Free the 2-d memory allocated to U.

Output

```
Enter dimension (n): 5
  0  -1  -2  -3  -4
  1   0  -1  -2  -3
  2   1   0  -1  -2
  3   2   1   0  -1
  4   3   2   1   0
```

Four types of 2-d arrays

Declaration	Number of rows	Number of columns
`int A[10][20];`	Static	Static
`int (*p)[20];`	Dynamic	Static
`int *B[10];`	Static	Dynamic
`int **q;`	Dynamic	Dynamic