Tutorial 5
Language and Automata Theory

Finite Automata and Regular Languages

1. Consider the following language over the alphabet $\Sigma = \{a, b\}$:

$$L_1 = \Big\{x \in \{a, b\}^* \mid x \text{ starts with } ab \text{ but does not end with } ab\Big\}.$$

   (a) Write a regular expression for L_1.

   (b) Design a DFA for L_1.

2. Construct a regular expression over the alphabet $\Sigma = \{a, b, c\}$ for:

$$L_2 = \Big\{x \in \{a, b, c\}^* \mid x \text{ has } (4i+1)\ b\text{'s for some integer } i \geq 0\Big\}.$$

   Design a DFA for L_2. Is your DFA having the minimum number of states?

3. The language $L_3 = \{uvv^{\mathbf{R}}w \mid u, v, w \in \{a, b\}^+\}$ is regular. Here $v^{\mathbf{R}}$ is the reverse of v. Answer the following questions.

   (a) Design a regular expression whose language is L_3.

   (b) Convert the regular expression of Part-(a) to an equivalent NFA.

   (c) Convert the NFA in Part-(b) into an equivalent DFA using *subset-construction procedure*.

   (d) Optimize the number of states of the DFA obtained in Part-(c) to produce the minimum state DFA.

4. Consider the language $L_4 = \{x \in \{a, b\}^* \mid x \text{ ends with 3 consecutive } b\text{'s}\}$.

   Answer the following.

   (a) Design an NFA for L_4.

   (b) Using subset construction, construct an equivalent DFA for the NFA from Part-(a).

   (c) Reduce the number of states in the resulting DFA (by removing the unreachable states and merging all the equivalent states).

5. Use pumping lemma to prove that the following languages are non-regular:

   - $L_{5a} = \{a^{k^3} \mid k \geq 0\}$.
   - $L_{5b} = \{a^{n!} \mid n \geq 0\}$.
   - $L_{5c} = \{a^p \mid p \text{ is prime}\}$.
   - $L_{5d} = \{a^i b^j a^{ij} \mid i, j \geq 0\}$.

6. Determine the regularity/non-regularity of the following languages:

   - $L_{6a} = \Big\{x \in \{a, b\}^* \mid \#a(x) - \#b(x) = 2024\Big\}$.
   - $L_{6b} = \Big\{x \in \{a, b\}^* \mid \#a(x) - \#b(x) \text{ is a multiple of } 2024\Big\}$.

   where, $\#a(x)$ and $\#b(x)$ denote the number of a's and b's in x.

   [*Hint:* Instead of finding regular expressions or using pumping lemma, you may alternatively try using the Myhill-Nerode theorem and show whether the number of equivalent classes formed is finite/infinite.]

7. Design a DFA for the following language:

$$L_7 = \left\{x \in \{a, b\}^* \mid \#a(x) = 0 \ (mod\ 2) \text{ and } \#b(x) = 0 \ (mod\ 3)\right\}.$$

where, $\#a(x)$ and $\#b(x)$ denote the number of a's and b's in x.

8. Let A, B be languages over an alphabet Σ, and $C = A \setminus B$. Then, justify which of the following statements is/are true?

   (a) If A and B are regular, then C is regular.

   (b) If A and C are regular, then B is regular.

   (c) If B and C are regular, then A is regular.

   (d) If C is regular, then A and B are regular.

9. Two regular expressions over the same alphabet are called *equivalent* if they generate the same language. Prove/Disprove the equivalence of the following pairs of regular expressions over the alphabet $\{a, b\}$.

   (a) $(ab + a)^*a$ and $a(ba + a)^*$.

   (b) $(ab^*a + ba^*b)^*$ and $(ab^*a)^* + (ba^*b)^*$.

10. For a string x, $x^{\mathbf{R}}$ denotes the reverse of x. Define $A^{\mathbf{R}} = \left\{x^{\mathbf{R}} \mid x \in A\right\}$ for a language A. Prove or disprove: if A is regular then so is $A^{\mathbf{R}}$.

11. Give regular expressions for each of the following languages of $\{a, b\}^*$.

    (a) $X = \left\{x \mid x \text{ contains an odd number of } a\text{'s}\right\}$.

    (b) $Y = \left\{y \mid y \text{ contains an even number of } b\text{'s}\right\}$.

    (c) $Z = \left\{z \mid z \text{ contains an odd number of } a\text{'s and an even number of } b\text{'s}\right\}$.

12. Which of the following are regular languages? Justify.

    (a) $U_1 = \left\{x \in \{a, b\}^* \mid x \text{ does not have three consecutive occurrences of } a\right\}$.

    (b) $U_2 = \left\{x \in \{a, b\}^* \mid x = x^{\mathbf{R}}\right\}$. Note that, $x^{\mathbf{R}}$ denotes the reverse of x.

    (c) $U_3 = \left\{x \in \{a, b, c\}^* \mid \#b(x) = 4n + 1,\ n \geq 1\right\}$. Note that, $\#b(x)$ denotes number of b's in x.
