INDIAN INSTITUTE OF TECHNOLOGY KHARAGPUR

Stamp / Signature of the Invigilator

EXAMINATION (Mid Semester)	**SEMESTER (Autumn 2024-2025)**

Roll Number										**Section**		**Name**	

Subject Number	C	S	6	0	0	0	5	**Subject Name**	*FOUNDATIONS OF COMPUTING SCIENCE*

Department / Center of the Student		**Additional sheets**	

Important Instructions and Guidelines for Students

1. You must occupy your seat as per the Examination Schedule/Sitting Plan.
2. Do not keep mobile phones or any similar electronic gadgets with you even in the switched off mode.
3. Loose papers, class notes, books or any such materials must not be in your possession, even if they are irrelevant to the subject you are taking examination.
4. Data book, codes, graph papers, relevant standard tables/charts or any other materials are allowed only when instructed by the paper-setter.
5. Use of instrument box, pencil box and non-programmable calculator is allowed during the examination. However, exchange of these items or any other papers (including question papers) is not permitted.
6. Write on both sides of the answer script and do not tear off any page. **Use last page(s) of the answer script for rough work.** Report to the invigilator if the answer script has torn or distorted page(s).
7. It is your responsibility to ensure that you have signed the Attendance Sheet. Keep your Admit Card/Identity Card on the desk for checking by the invigilator.
8. You may leave the examination hall for wash room or for drinking water for a very short period. Record your absence from the Examination Hall in the register provided. Smoking and the consumption of any kind of beverages are strictly prohibited inside the Examination Hall.
9. Do not leave the Examination Hall without submitting your answer script to the invigilator. **In any case, you are not allowed to take away the answer script with you.** After the completion of the examination, do not leave the seat until the invigilators collect all the answer scripts.
10. During the examination, either inside or outside the Examination Hall, gathering information from any kind of sources or exchanging information with others or any such attempt will be treated as **'unfair means'**. Do not adopt unfair means and do not indulge in unseemly behavior.

Violation of any of the above instructions may lead to severe punishment.

Signature of the Student

To be filled in by the examiner

Question Number	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**Total**
Marks Obtained											

Marks obtained (in words)	**Signature of the Examiner**	**Signature of the Scrutineer**

Indian Institute of Technology Kharagpur
Department of Computer Science and Engineering

Foundation of Computing Science (CS60005) **Autumn Semester 2024-2025**

19-September-2024 **Mid-Semester Examination** Maximum Marks: 60

Instructions:

- Write your answers in the question paper itself. Be brief and precise. Answer *all* questions.
- Write the answers only in the respective spaces provided. The last two blank pages may be used for rough work or leftover answers.
- In case you may need more space/pages, please ask for additional sheets in the exam hall and attach the same with this booklet while submitting.
- If you use any theorem / result / formula covered in the class, just mention it, do not elaborate. (unless the same thing has been explicitly asked to derive / prove in the question)
- Write all the proofs in mathematically / logically precise language. Unclear and/or dubious statements would be severely penalized.

— Question Paper Starts from Next Page —

Q1. Encode the five English statements given below, into well-formed predicate-logic formulas. Your encodings should use ***only*** the following predicates with the given meanings. **(2 × 5)**

$\text{boy}(x)$	: x is a boy
$\text{girl}(x)$	: x is a girl
$\text{love}(x,y)$	: x loves y
$\text{marry}(x,y)$	: x marries y
$\text{diff}(x,y)$	: x and y are different

(a) Every boy who loves a girl does not love every other boy whom that girl loves.
Solution:

(b) Every boy who loves a girl marries that girl, but not every girl who marries a boy loves that boy.
Solution:

(c) Everyone loves either himself/herself or everyone else except himself/herself.

Solution:

(d) Every girl loves exactly one boy.

Solution:

(e) Every boy loves at least two girls.

Solution:

Q2. Suppose, ρ and σ are two binary relations defined over the set $\mathscr{A}$. A *composite relation* $\rho \circ \sigma$ over $\mathscr{A}$ is defined as, $\rho \circ \sigma = \{(p,r) \mid$ there exists some $q \in \mathscr{A}$, such that $(p,q) \in \rho$ and $(q,r) \in \sigma\}$.

Prove the following assertions with precise formal justifications.

(a) If ρ and σ are two equivalence relations over $\mathscr{A}$, then prove that $\rho \circ \sigma$ is an equivalence relation *if and only if* $\rho \circ \sigma = \sigma \circ \rho$. **(7)**

Solution:

(b) The *inverse* of a relation τ over $\mathscr{A}$ is defined as, $\tau^{-1} = \{(q,p) \mid (p,q) \in \tau \text{ and } p,q \in \mathscr{A}\}$. Prove that, $(\rho \circ \sigma)^{-1} = (\sigma^{-1} \circ \rho^{-1})$. **(4)**

Solution:

Q3. **(a)** Let $(G, \circ)$ be a group and $c \in G$. Define a binary composition $*$ on G by $a * b = a \circ c \circ b$ for all $a, b \in G$. Show that $(G, *)$ is a group, clearly indicating all the properties of a group. **(4)**

Solution:

(b) Let G be a multiplicative group. Prove that, if $(ab)^2 = a^2b^2$ for all $a, b \in G$, then G is abelian. **(3)**

Solution:

Q4. **(a)** Suppose that L_1 and L_2 are two languages (over the same alphabet) given to you such that L_1 and L_1L_2 (concatenation) are both regular. Prove or disprove: L_2 must be regular too. **(3)**

Solution:

(b) Consider the language, $L_3 = \{a^i b^j \mid i, j \geq 0 \text{ and } |i-j| \text{ is a prime}\}$. (Note that, 1 is not a prime.) Using the pumping lemma for regular languages, prove that the language L_3 is *not* regular. **(4)**

Solution:

(c) Consider the language, $L_4 = \{\alpha \in \{a,b\}^* \mid |\alpha| = n^2 \text{ for some integer } n \geq 0\}$, where $|\alpha|$ denotes the length of the string α. Using the pumping lemma for context-free languages, prove that the language L_4 is *not* context-free. **(4)**

Solution:

Q5. Consider the language, $L_5 = \{\alpha \in \{0,1,2\}^* \mid \alpha \text{ does not contain two consecutive 0's}\}$.

(a) Describe a regular grammar for L_5. **(4)**

Solution:

(b) Design a deterministic finite automaton (DFA) to accept L_5. **(3)**

Solution:

Q6. Consider the language, $L_6 = \left\{ a^{3k+1} b^{5k-2} \mid k \geq 1 \right\} \subseteq \{a, b\}^*$.

(a) Write a context-free grammar (CFG) G with $\mathscr{L}(G) = L_6$. **(4)**

Solution:

(b) Design a push-down automaton (PDA) M with $\mathscr{L}(M) = L_6$.

[

Hint: You may use CFG-to-PDA conversion procedure.] **(4)**

Solution:

Q7. **(a)** Use a diagonalization argument to prove that the set of all infinite sequences of natural numbers is *uncountable*. **(3)**

Solution:

(b) Prove that the set of all finite sequences of natural numbers is *countable*. **(3)**

Solution:

— Question Paper Ends Here —