# CS60005 : Foundations of Computing Science (Autumn 2024-2025)

## Class Test 1

27-Aug-2024 (Tuesday) Maximum Marks: 20 06:00pm – 07:00pm

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**Roll:** ____________________ **Name:** ______________________________________________

[ *Write your answers in question paper. Answer all questions. Be brief and mathematically / logically precise.* ]

**Q1.** Let $z_1,\ z_2,\ \ldots,\ z_n$ be positive integers with $\sum_{i=1}^{n} z_i < 2^n - 1$. Prove that, there exist distinct disjoint non-empty subsets $A$, $B$ of the set $\{z_1,\ z_2,\ \ldots,\ z_n\}$ with the property that $\sum_{a \in A} a = \sum_{b \in B} b$. **(4)**

**Solution:**

**Q2.** From the semester examination results, the departmental head makes the following observations.

$F_1$ : *If Alice is meritorius, then Bob is not studious or Charles is not attentive.*

$F_2$ : *If Charles is attentive, then Diana is sincere.*

$F_3$ : *If Diana is sincere and Bob is studious, then Alice is meritorius.*

$F_4$ : *Bob is studious.*

You task is to answer the question: *"Is Charles attentive?"*

Frame (encode) the arguments logically, and formally deduce (applying logical inferencing rules) the answer being asked here. Present your solution as indicated in the following parts.

**(a)** Write all the propositions (that you will use to encode) with English statements (meaning). **(2)**

**Solution:**

**(b)** Write propositional logic formula to encode each of the *four* statements $F_1 - F_4$ given above. **(2)**

**Solution:**

– Question **Q2** continued to next page –

**(c)** Use logical inferencing rules to completely (and step-wise) derive the answer, and finally conclude whether Charles is attentive or not. **(4)**

**Solution:**

**Q3.** Let $\mathbb{A}$ be one of the sets $\mathbb{N}$ (the set of natural numbers including zero), $\mathbb{Z}$ (the set of integers), $\mathbb{Q}$ (the set of rationals), or $\mathbb{R}$ (the set of reals). A function $\varphi : \mathbb{A} \to \mathbb{A}$ is called *monotonic increasing* if $m \leq n$ implies $\varphi(m) \leq \varphi(n)$. It is called *strictly monotonic increasing* if $m < n$ implies $\varphi(m) < \varphi(n)$.

**(a)** Show that a strictly monotonic increasing function is injective. **(2)**

**Solution:**

– Question **Q3** continued to next page –

**(b)** Give an example of a function $\mathbb{N} \to \mathbb{N}$ that is injective but not strictly monotonic increasing. **(2)**

**Solution:**

**(c)** Let $f : \mathbb{N} \to \mathbb{N}$ be an injective function with the property that $\left| f(n+1) - f(n) \right| \leq 1$ for all $n \in \mathbb{N}$. Show that $f$ is strictly monotonic increasing. **(4)**

**Solution:**

— End of Question Paper —