TUTORIAL – 5
(CONTEXT-FREE LANGUAGES & PUSH-DOWN AUTOMATA)

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Problem-1

What context-free languages will be generated by the following two (separate) CFGs,

$G = < \{S, A, B\}, \{a, b, c\}, P, S >$,

where P consists of the following production rules?

(a) $S \rightarrow ASB|\varepsilon$, $A \rightarrow a$, $B \rightarrow bb|b$

(b) $S \rightarrow abScB|\varepsilon$, $B \rightarrow bB|b$

Problem-2

Define the context-free grammars for the following context-free languages?

Are your defined CFGs ambiguous / non-ambiguous?

- $L = \{ a^i b^j c^k \mid i,j,k \geq 0, \text{ and } i=j \text{ or } i=k \}$
- $L' = \{ a^i b^j c^k \mid i,j,k \geq 0, \text{ and } i+j = k \}$

Problem-3

Which of the following language(s) is/are context-free?

Give justifications.

- $L_1 = \{ a^m b^n \mid m,n \geq 0, m = 2n \}$
- $L_2 = \{ a^m b^n \mid m,n \geq 0, m \neq 2n \}$
- $L_3 = \{ a^m b^m c^{m+n} \mid m,n \geq 1 \}$
- $L_4 = \{ a^m b^n c^{m+n} \mid m,n \geq 1 \}$
- $L_5 = \{ a^l b^m c^n \mid l \geq 0, l < m \text{ and } l < n \}$

Problem-4

Consider the two CFGs G and G' with the start symbols S and S' and with the only productions:

Productions of G : $S \rightarrow aS \mid B, \quad B \rightarrow bB \mid b$

Productions of G': $S' \rightarrow aA' \mid bB', A' \rightarrow aA' \mid B', B' \rightarrow bB' \mid \varepsilon$

Prove that, $L(G) \subset L(G')$, i.e., L(G) is strictly contained in L(G').

Problem-5

Prove that, the following context-free grammar,

G = ({S}, {a, b, c}, P, S), is ambiguous.

Here, the production rules (P) are given as:

$$S \rightarrow aS \mid aSbS \mid c.$$

Construct a non-ambiguous grammar, G', that derives the same language.

Also prove L(G) = L(G').

Problem-6

Given the following languages over the alphabet {a, b}, design PDAs that accepts by empty stack (separate PDA for each one).

- L = (a+b)*b
- L' = all palindromes over {a,b}

Since L is a regular languages, can you directly present the left linear and the right linear grammar for L and then formally derive the NFAs from these grammars?

THANK YOU !

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