TUTORIAL – 3 (COUNTABILITY & ALGEBRAIC STRUCTURES)

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Problem-1

Let A and B be uncountable sets with $A \subseteq B$.

Prove or disprove: A and B are equinumerous.

Problem-2

Let A be an uncountable set and B a countably infinite subset of A.

Prove or disprove: A is equinumerous with A – B.

Problem-3

Prove that the real interval $[0, 1)$ is equinumerous with the unit square $[0, 1) \times [0, 1)$.

Problem-4

Let Z[x] denote the set of all univariate polynomials with integer coefficients.

Prove that Z[x] is countable.

Problem-5

Define an operation $\circ$ on $G = R^* \times R$ as

$$(a,b)\circ(c,d)=(ac,\ bc+d).$$

Prove that, $(G,\circ)$ is a non-abelian group.

Problem-6

Let G be a (multiplicative) group, and H, K are subgroups of G.

Prove that,

(a) $H \cap K$ is a subgroup of G.

(b) $H \cup K$ is a subgroup of G if and only if $H \subseteq K$ or $K \subseteq H$.

(c) Define $HK = \{hk \mid h \in H, k \in K\}$. Define KH analogously. Prove that, HK is a subgroup of G if and only if $HK = KH$.

Problem-7

Let $R = Z \times Z$, and r, s be constant integers.

Define two operations, on R as follows:

$$(a,b)+(c,d) = (a+c,b+d) \quad \text{and}$$

$$(a,b)*(c,d) = (ad+bc+rac,bd+sac).$$

Prove that, R is a ring under these operations, $+$ and $*$.

Problem-8

Let R_1, R_2, ..., R_n be rings.

Prove that, the Cartesian product $(R_1 \times R_2 \times \cdots \times R_n)$ is a ring under component-wise addition and multiplication.

Show that, if each R_k is a ring with identity, then so also is the product.

THANK YOU !

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