TUTORIAL – 2
(SET, RELATION, FUNCTION)

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Problem-1

Let A, B, C $\in$ U are three arbitrary sets such that

$$A \cup B = A \cup C \quad \text{and} \quad A \cap B = A \cap C.$$

Prove that, $B = C$.

Problem-2

For a function, $f : A \longrightarrow B$, define a function $\mathcal{F} : \mathcal{P}(A) \longrightarrow \mathcal{P}(B)$ as $\mathcal{F}(S) = f(S)$ for all $S \subseteq A$.

Prove that:

(a) $\mathcal{F}$ is injective if and only if f is injective.

(b) $\mathcal{F}$ is surjective if and only if f is surjective.

Problem-3

Let $f : A \longrightarrow B$ be a function and σ an equivalence relation on B.

Define a relation ρ on A as: $a \,\rho\, a'$ if and only if $f(a) \,\sigma\, f(a')$.

Answer the following:

(a) Prove that, ρ is an equivalence relation on A.

(b) Define a map $\bar{f} : A/\rho \longrightarrow B/\sigma$ as $[a]_\rho \longmapsto [f(a)]_\sigma$.
Prove that, $\bar{f}$ is well-defined.

(c) Prove that, $\bar{f}$ is injective.

(d) Prove or disprove: If f is a bijection, then so also is $\bar{f}$.

(e) Prove or disprove: If $\bar{f}$ is a bijection, then so also is f.

Problem-4

[Genesis of rational numbers]

Define a relation ρ on $A = Z \times (Z \setminus \{0\})$ as $(a, b)\ \rho\ (c, d)$ if and only if $ad = bc$. (Here, Z is the set of integers)

(a) Prove that ρ is an equivalence relation.

(b) Argue that A/ρ is essentially the set Q of rational numbers.

Problem-5

Let ρ be a total order on A. We call ρ a well-ordering of A if every non-empty subset of A contains a least element. In this exercise, we plan to construct a well-ordering of $A = N \times N$. (Here, N is the set of natural numbers)

(a) Define a relation ρ on A as
 $(a,b)\ \rho\ (c,d)$ if and only if $a \leq c$ or $b \leq d$.

(b) Define a relation σ on A as
 $(a,b)\ \sigma\ (c,d)$if and only if $a \leq c$ and $b \leq d$.

(c) Define a relation $\leq_L$ on A as
 $(a,b) \leq_L (c,d)$ if either (i) $a < c$, or (ii) $a = c$ and $b \leq d$.

Prove or disprove: ρ, σ, $\leq_L$ is a well-ordering of A.

THANK YOU !

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