Indian Institute of Technology Kharagpur
Department of Computer Science and Engineering

Foundations of Computing Science (CS60005)	**Autumn Semester, 2022-2023**
Class Test 2 **Date: 05-Nov-2022 (Saturday) 10:30AM − 11:30AM**	**Marks: 20**

Instructions:

- There are THREE questions. Answer ALL questions.

- Write your answers in the answer booklet provided to you in the examination hall.

- Keep you answers brief and precise. Write solutions for all parts of a question together.

- Precisel state all assumptions you make.

- Sketchy proofs and claims without proper reasoning will be given no credit.

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1. Prove or disprove the following statements.

    (a) Every infinite regular set contains a subset that is not recursively enumerable. `4`

    (b) Every infinite $r.e.$ set contains an infinite recursive subset.

    Hint: A set is recursive iff there exists an enumeration machine enumerating its strings in non-decreasing order of length. `6`

2. Consider the languge $\{(\mathcal{M}, x, p) \mid \mathcal{M}$ on input x visits state p during the computation$\}$. (Here, $p \in Q$ with Q being the set of states of $\mathcal{M}$ and $x \in \Sigma^*$ where Σ is the input alphabet of the Turing machine $\mathcal{M}$.) Is this language decidable? Justify. `5`

3. Let $\mathsf{REG} = \{\mathcal{M} \mid \mathcal{M}$ is a TM and $L(\mathcal{M})$ is a regular set$\}$. One of the following is true. Identify which one and justify your answer. `5`

    (a) REG is recursive.

    (b) REG is $r.e.$ and $\neg\mathsf{REG}$ is not $r.e.$

    (c) REG is not $r.e.$ and $\neg\mathsf{REG}$ is $r.e.$

    (d) Neither REG nor $\neg\mathsf{REG}$ is $r.e.$