
Indian Institute of Technology Kharagpur
Department of Computer Science and Engineering

Foundations of Computing Science (CS60005)

Autumn Semester, 2022-2023

Class Test 1

01-Sep-2022 (Thursday), 17:30–18:30

Maximum Marks: 20

Instructions:

- Write your answers in the answer booklet provided to you in the examination hall.
 - There are a total of **THREE** questions, having 6 marks, 7 marks and 7 marks, respectively.
 - Answer **ALL** the questions (or as many as you can) mentioning the question numbers clearly.
 - Be brief and precise. Write the answers for all parts of a question together.
 - If you use any theorem/result/formula covered in the class, just mention it, do not elaborate.
 - Write all the proofs/deductions in mathematically/logically precise language.
Unclear and/or dubious statements would be severely penalized.
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Q1. Let $\text{Cat}(x)$, $\text{Dog}(x)$, $\text{Striped}(x)$, $\text{Friends}(x, y)$, $\text{Equal}(x, y)$ and $\text{ShortTempered}(x)$ be predicates to be evaluated over the set of all animals. The predicates have the following obvious interpretations.

- $\text{Cat}(x)$ (or $\text{Dog}(x)$) evaluates to True iff x is a cat (or dog).
- $\text{Striped}(x)$ evaluates to True iff the coat of x is striped.
- $\text{Friends}(x, y)$ evaluates to True iff x and y are friends.
Obviously, $\text{Friends}(x, y)$ also implies $\text{Friends}(y, x)$.
- $\text{Equal}(x, y)$ evaluates to True iff x and y are one and the same animal.
- $\text{ShortTempered}(x)$ evaluates to True iff x is short-tempered.

Express the following English language sentences as predicate logic sentences (formulae without free variables). You may assume that the domain for evaluating the truth of these sentences is always the set of all animals (which could include animals other than cats and dogs as well).

- (a) There is a short-tempered dog who is not friendly with any other dog, but is friendly with at least one striped cat. (2)
- (b) Every cat that is not striped is friendly with at least one dog that is not a friend of any striped cat. (2)
- (c) Every short-tempered cat is friendly with one and only one striped dog. (2)

Q2. Let \mathbb{Z} be the set of all integers. Define a relation R on \mathbb{N} (the set of positive integers) as follows:

$$\forall a, b \in \mathbb{N}, a R b \text{ if and only if } \exists i \in \mathbb{Z}, \frac{a}{b} = 2^i.$$

- (a) Prove that R is an equivalence relation. (3)
- (b) List the equivalence classes defined by R on \mathbb{N} . (2)
- (c) Prove / Disprove: R is a partial order. (2)

Q3. Consider the real intervals $(0, 1)$ and $[0, 1]$. A function $f : (0, 1) \rightarrow [0, 1]$ is defined as follows.

Take $x \in (0, 1)$. Find (the unique) $n \in \mathbb{N}$ such that $\frac{1}{2^n} \leq x < \frac{1}{2^{n-1}}$. Define $f(x) = \frac{3-2^n x}{2^n}$.

- (a) Prove / Disprove: f is injective. (4)
 - (b) Prove / Disprove: f is surjective. (3)
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