TUTORIAL 9

① $L \in PSPACE$ — N : DTM (using poly space) deciding L.

$L^* \in PSPACE$

$x \in L^*$ if $x = \varepsilon$ or

$x \in L$ or

$x = wz$ where $w \in L$ & $z \in L^*$.

$PSPACE = NPSPACE$

NDTM M to check if $x \in L^*$.

- Guess w. Check that $w \in L$ using N.
- Recursively check if $z \in L^*$.

DTM deciding membership in L^*: use dynamic programming.

$x = x_1 x_2 \dots x_n$ $T = (t_{ij})$ $t_{ij} = 1$ if $x_{i,j} \in L^*$ & 0 o.w.

$x_{i,j} = x_i x_{i+1} \cdots x_j$

$x \in L^* \iff t_{1,n} = 1$

(2) (M, s, t)

reject if $|s| \neq |t|$

Σ : alphabet of M.

Construct a graph G with vertices $\Sigma^{|s|}$.

$\{w, x\} \in E(G)$ if x can be obtained by replacing one symbol of w.

(s, t) defines a ladder if $\exists$ a path from s to t & all words of this path are accepted by M.

Can be checked in Polyspace!

④ (a) Savitch's thm $\overset{?}{\Rightarrow}$ PATH $\in$ SC.

↳ named after Stephen Cook

$NSPACE(S(n)) \subseteq DSPACE(S(n)^2)$

SC : class of all languages decidable in deterministic poly time + polylog space.

PATH $\in$ NL. $\overset{\text{Savitch}}{\Rightarrow}$ PATH $\in$ DSPACE$(\log^2 n)$

$DSPACE(\log^2 n) \subseteq DTIME(2^{\log^2 n})$

$= DTIME(n^{\log n})$ not poly time.

$\exists$ a poly-time DTM deciding PATH
$\exists$ a poly-space DTM deciding PATH
$\Rightarrow$ $\exists$ a single DTM using poly time & polylog space deciding PATH.

(b) $SC \subseteq P \cap polyL$