TUTORIAL 8

3. $P = coP$ $\qquad coP = \{ L : \neg L \in P \}$

$L \in P \qquad \neg L \in coP$

M : poly-time DTM deciding L.

N : DTM that simulates M & flips its decision

N : poly-time decider for $\neg L$.

$\Rightarrow$ $coP \subseteq P$

$P \subseteq coP$

$\Rightarrow P = coP$.

$P \subseteq NP \cap coNP$

We know $P \subseteq NP$.

$L \in NP$ iff $\neg L \in coNP$.

Let $L \in P$. Then $\neg L \in coP = P \subseteq NP$. ie, $\neg L \in NP$.

By defn. of coNP, $\neg\neg L = L \in coNP$.

$\Rightarrow P \subseteq coNP$.

5. HP is NP-Hard.

$SAT \leq_p HP$

$\phi \longmapsto (N, \phi)$

$SAT \in NP$ — poly-time DTM M s.t. $\forall \phi$, $\phi \in SAT$ iff $\exists z$ s.t. $M(\phi, z) = 1$

N: on i/p y

- parse y as a Boolean formula over n variables (no. of variables in ϕ)
- $\forall z \in \{0,1\}^n$, check if $M(y, z)$ accepts. If M accepts, accept & halt.
- Enter a trivial loop.

Description of N can be written down in poly-time. (poly in $|\phi|$)
(considering parameters in M's description as constants)

$\phi \in SAT \Rightarrow N$ on i/p ϕ will halt $\Rightarrow (N, \phi) \in HP$

$\phi \notin SAT \Rightarrow$ N on i/p ϕ will loop

$\Rightarrow (N, \phi) \notin HP.$

$SAT \in NP\text{-Hard} \Rightarrow HP \in NP\text{-Hard}$

6. DOUBLESAT $\in$ NP Certificate is 2 satisfying assignments.

DOUBLESAT $\in$ NP-Hard

$SAT \leq_p$ DOUBLESAT

$\phi \longmapsto \underbrace{\phi \wedge (x \vee \neg x)}_{\phi'}$ x is a new variable

β is a satisfying assignment for ϕ

Then $(\beta, 1)$ & $(\beta, 0)$ are both satisfying assignments for ϕ'.