TUTORIAL 7

1) $A = \{ M \mid M$ is a DFA not accepting any string with odd 1's $\}$

$E\text{-}DFA = \{ M \mid M$ is a DFA & $L(M) = \phi \}$ is decidable

Check if final states are reachable from the start state. If not, then $L(M) = \phi$.

N : TM deciding $E\text{-}DFA$.

$ODD = \{ x \mid x$ contains odd no. of 1's $\}$.

$M \in A$ iff $L(M) \cap ODD = \phi$.

ODD is regular. M_{ODD} : DFA accepting ODD.

Checking membership of $M \in A$: Construct DFA M' accepting $L(M) \cap ODD$. Use N to determine whether $L(M') = \phi$.

If $L(M') = \phi$, then $M \in A$; o.w. $M \notin A$.

3) $\text{EQUIV} = \{(M_1, M_2) \mid L(M_1) = L(M_2)\}$

$\text{HP} \leq_m \text{EQUIV}$ HP is undecidable $\Rightarrow$ EQUIV is undecidable

$(M, x) \mapsto (M_1, M_2)$

M_1 : on i/p y

- erase y
- accept

$L(M_1) = \Sigma^*$

M_2 : on i/p y

- erase y
- run M on x
- accept if M halts.

$$L(M_2) = \begin{cases} \Sigma^* & \text{if } (M, x) \in HP \\ \phi & \text{if } (M, x) \notin HP \end{cases}$$

$(M, x) \in HP \Rightarrow L(M_1) = L(M_2)$

$(M, x) \notin HP \Rightarrow L(M_1) \neq L(M_2)$

5) $B = \{ M \mid L(M)$ contains ≤ 300 elements$\}$

non-montone property

$\therefore$ by Rice's thm part II, B is not r.e.

6) a) $P_{REG}(A) = \begin{cases} T & \text{if } A \text{ is regular} \\ F & \text{o.w} \end{cases}$

$A = \{01\}$

$B = \{0^n 1^n : n \geq 1\}$

$\downarrow$

non-monotone property & hence by Rice's thm is not r.e.

$P_{\neg REG}(A) = \begin{cases} T & \text{if } A \text{ is not regular} \\ F & \text{o.w} \end{cases}$

$A = \{0^n 1^n : n \geq 1\}$

$B = \{0,1\}^*$

$\downarrow$

non-monotone

b) $TOT = \{ M \mid M \text{ halts on all inputs} \}$

TOT is not r.e.

$\neg HP \leq_m TOT$

$(M, x) \longmapsto N$

N : on i/p y
- run M on x for $|y|$ steps
- halt if M does not halt within $|y|$ steps.
- O.W., enter a trivial loop.

$(M, x) \in \neg HP$ i.e., M does not halt on x

$\Rightarrow N$ halts on all inputs i.e, $N \in TOT$

$(M, x) \notin \neg HP$ i.e., M halts on x in, say, n steps

$\Rightarrow N$ loops on all strings from $\Sigma^{\geq n} \Rightarrow N \notin TOT$

7) K : TM deciding MP.

$MP = \{(M, x) \mid x \in L(M)\}$

Can we use K to prove or disprove Collatz conjecture?

N_1 : on i/p x

- while $x \neq 1$

  if x is even, $x \leftarrow x/2$

  o.w $x \leftarrow 3x+1$

- accept

N_2 : on i/p y

- erase y
- $x \leftarrow 1$

repeat forever {
- run K on (N_1, x)
- if K rejects, accept
- $x \leftarrow x+1$

If conjecture is false, the N_2 halts & accepts after finding a counter example.

O.w. N_2 runs forever.

Use K to determine whether N_2 accepts an arbitrary i/p y or not.

If K accepts then conjecture is false o.w, conjecture is true.