Let A,B be languages over an alphabet Σ, and C = A–B.
Which of the following statements must be true?

(A) If A and B are regular, then C is regular.
(B) If A and C are regular, then B is regular.
(C) If B and C are regular, then A is regular.
(D) If C is regular, then A and B are regular.

(A) $C = A - B = A \cap \overline{B}$ ← regular [Regular is closed under intersection]

↳ regular

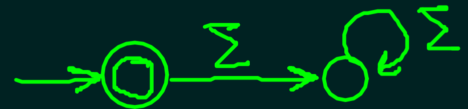
↳ TRUE

(B) FALSE

$B = \{a^n b^n \mid n \geq 0\}$

$C = \emptyset$ $A = \{\epsilon\}$

DFA(A) :



DFA(C) :

~~$A = (a+b)^*$ $B = \{a^n b^n \mid n \geq 0\}$ $C = A - B$ ← ~~not~~ regular~~ ✗

$A = \mathcal{L}(b^* a^*)$ $B = \{a^n b^n \mid n \geq 0\}$ $C = \mathcal{L}(a^+ b^+)$

(C) FALSE $A = \{w w^{rev} \mid w \in \{a,b\}^*\}$ $B = (a+b)^*$ aaaa

w

$C = \emptyset$

$A = \{a^n b^n \mid n \geq 0\}$ $B = a^* b^*$ $C = \emptyset$

(D) FALSE $A = \{a^n b^n \mid n \geq 1\}$ $B = \{b^n a^n \mid n \geq 1\}$

$C = A - B = \emptyset$

Consider the following language over the alphabet {a,b}:

$L1 = \{ x \in \{a,b\}^* \mid x \text{ starts with ab but does not end with ab} \}$.

(A) Write a regular expression for L1. (B) Design a DFA for L1.

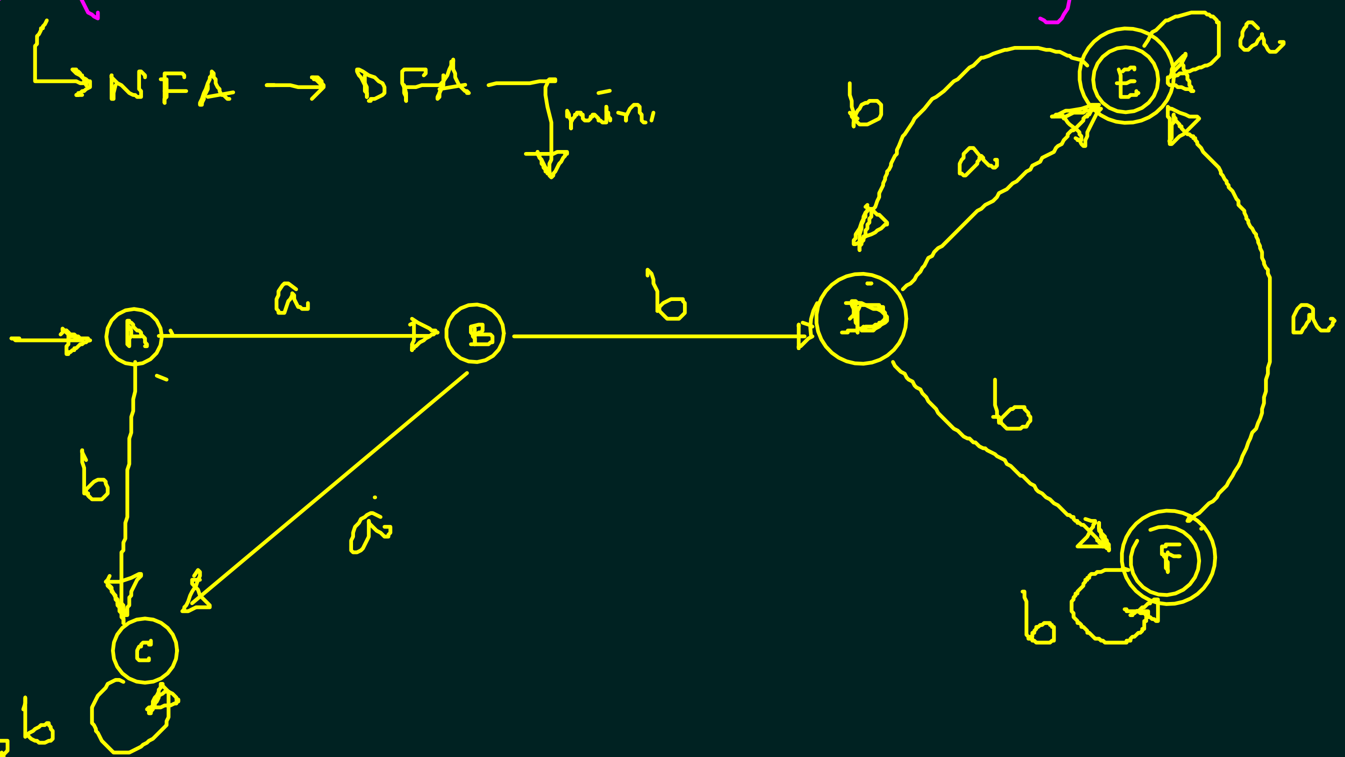
(A) $ab(a+b)^*(aa+ba+bb)$ + aba + abb

$ab \notin L_1$ ✗

$RE(L_1) = ab(a+b+(a+b)^*(aa+ba+bb))$

→ NFA → DFA → min.

(B)



The language L2 = { uvv'w | u,v,w $\in$ {a,b}+ } is regular. Here, v' is the reverse of v.

(a) Design a regular expression whose language is L2.

(b) Convert the regular expression of Part (a) to an equivalent NFA.

(c) Convert the NFA in Part (b) to an equivalent DFA.

(d) Minimize the number of states of the DFA obtained in Part (c).

HOMEWORK

(a) u v v' w

aa

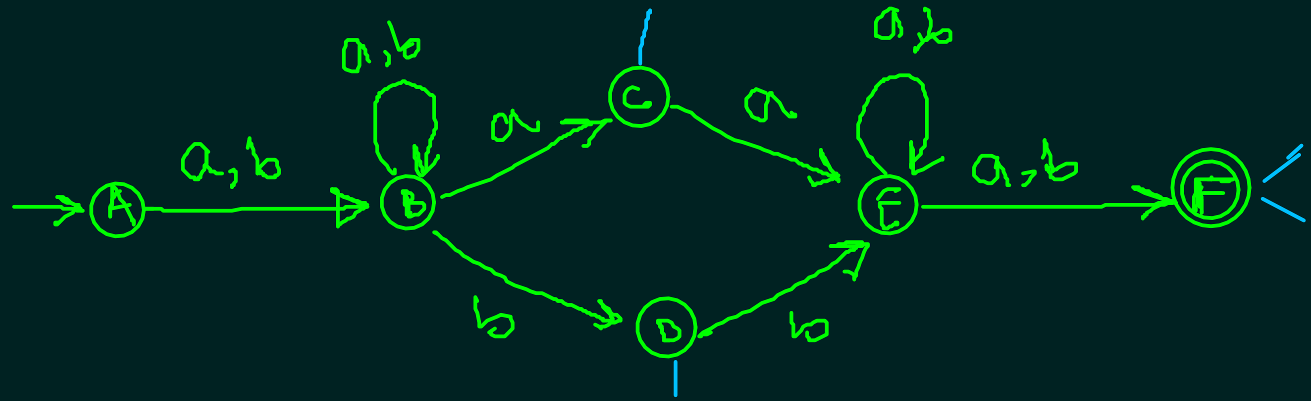
bb

$\in L_2$

RE = $(a+b)(a+b)^* (aa+bb)(a+b)(a+b)^*$

DFA

(b)



(C) Do yourself.

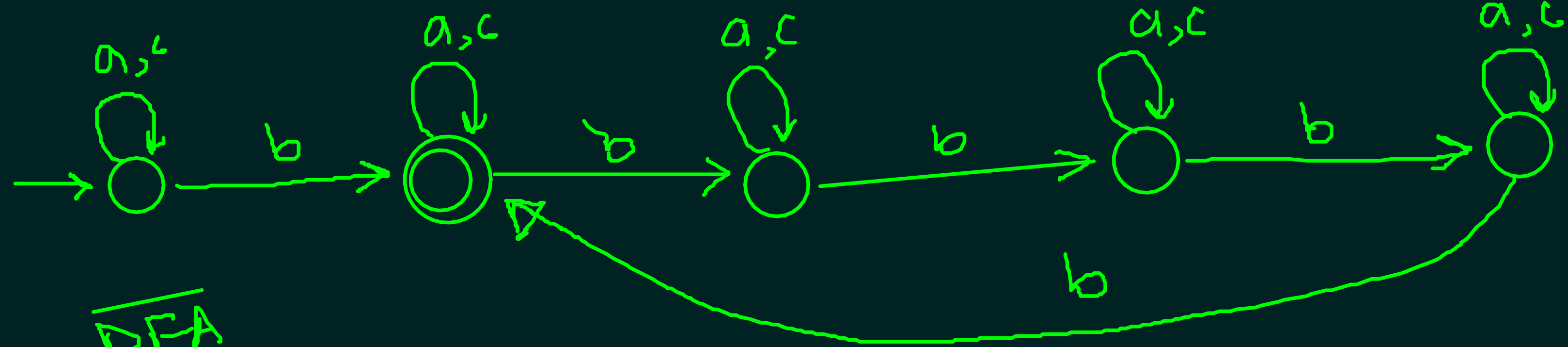
(d)

Construct a regular expression over the alphabet {a,b,c} for
L3 = { x ∈ {a,b, c}* | x has 4i+1 b's for some integer i >= 0 }.
Construct an NFA from it, then build the equivalent DFA and minimize.

RE(L3) =

\# b's → 1, 5, 9, 13

$$\left((a+c)^* b (a+c)^* b (a+c)^* b (a+c)^* b\right)^* (a+c)^* b (a+c)^*$$



minimum?

DFA

Use Pumping Lemma to prove that the following languages are not regular.

(A) L4 = { a^n! | n >= 0 }   (B) L5 = { a^p | p is prime }

(A) $\alpha = x y z$ ($x = \epsilon$, $z = \epsilon$) $y = a^{k!}$ accepted by DFA of $\underbrace{k}$ states

$= uvw$

$\longrightarrow v = a^{l}$ $(\underline{1 \le l \le k})$

$\exists i$: $uv^iw \notin L$

$\underline{i=2}$: $uv^2w = a^{k!+l} \notin L$ $k! < \underbrace{k!+l}_{\text{not a factorial term}} \le k!+k < (k+1)!$

(B) $y = uvw = a^{p}$ (p prime) where $v = a^{l}$ $p < \boxed{p+c} < 2p$

10 20

13 17 19

$uv^iw \notin L$

for which i?

$uv^pw = a^{p+pl} = a^{p(1+l)} \notin L$

a^{l}

a^{2l}

a^{3l}

?? $(p+c)$ is not prime

$$\boxed{\begin{array}{l} L_6 = \{0^{n^3} \mid n \ge 0\} \\ L_7 = \{a^{2^n} \mid n \ge 0\} \end{array}}$$

Two regular expressions over the same alphabet are called equivalent if they generate the same language. Prove/Disprove the equivalence of the following pairs of regular expressions over the alphabet {a,b}.

(A) $(ab+a)^*a$ and $a(ba+a)^*$ (B) $(ab^*a+ba^*b)^*$ and $(ab^*a)^*+(ba^*b)^*$

LHS RHS

Equiv

✓ NOT ✗ Counterexample string

RE_{LHS}

$$(ab+a)^*a = (a(b+\epsilon))^*a$$

A

$$= a(b+\epsilon)\,a\,(a)\,a(b+\epsilon)\,a$$

$$= a((b+\epsilon)a)^* = a(ba+a)^* = RE_{RHS}$$

B

aabb

bababa

Check

RE → NFA → DFA → Min