1. Which of the following sentences are valid (tautology), unsatisfiable, or neither.
   (a) (Smoke → Fire) → (¬Smoke → ¬Fire)
   (b) (Smoke → Fire) → ((Smoke ∧ Heat) → Fire)

(a) $\left.\begin{array}{l} \text{Smoke} \equiv \bot \\ \text{Fire} \equiv \bot \end{array}\right\}$ outcome $\equiv \top$ $\qquad$ $\left.\begin{array}{l} \text{Smoke} \equiv \bot \\ \text{Fire} \equiv \top \end{array}\right\}$ outcome $\equiv \bot$

(satisfiable) $\qquad$ (invalid)

(b) $\neg(\text{Smoke} \to \text{Fire}) \vee ((\text{Smoke} \wedge \text{Heat}) \to \text{Fire})$

$\equiv (\text{Smoke} \wedge \neg\text{Fire}) \vee (\neg\text{Smoke} \vee \neg\text{Heat} \vee \text{Fire})$

$\equiv (\underbrace{\text{Smoke} \vee \neg\text{Smoke}}_{\top} \vee \neg\text{Heat} \vee \text{Fire}) \wedge$

$(\neg\underbrace{\text{Fire} \vee \neg\text{Smoke} \vee \neg\text{Heat} \vee \text{Fire}}_{\top})$

$\equiv \top \wedge \top \equiv \top$ (valid) ✓

alternative : $\neg(b) \equiv \bot$ (You try) ✓

2. Prove the following logical deduction.

F_1: $(\neg p \vee q) \to r$ ≡ IFF, NNF

F_2: $r \to (s \vee t)$ ✓ ≡

F_3: $\neg (s \vee u)$

F_4: $t \to u$

F_5: $q \leftrightarrow v$

F_6: $(v \wedge \neg w) \vee (\neg v \wedge w) \to \neg p$

G: $\therefore \neg w$

① F_3

$\therefore \underbrace{\neg s}_{C_1}, \underbrace{\neg u}_{C_2}$

MT ② F_4, $\neg u$

$\therefore \neg t$

$a \to b \equiv (\neg a \vee b)$

$a \equiv (a \vee \bot)$

$\dfrac{a \quad\;}{b}$ $\quad b \vee \bot \equiv b$

$(a \vee b) \wedge (\neg a \vee c)$

$\Rightarrow b \vee c$

(Resolve)

$a \to b$

$\neg b$

$\neg a$

CNF

③ F_2

$\neg s, \neg t$

$\therefore \neg r$

④ F_1

$\neg r$

$\therefore \underbrace{p}_{c'}, \underbrace{\neg q}_{c''}$ ✓

⑤ F_6

p

$\therefore (\neg v \vee w) \wedge (v \vee \neg w)$

⑥ F_5

$\neg q$

$\therefore \neg v$

⑦ $v \vee \neg w$

$\neg v$

$\therefore \neg w$

$\neg (F_1 \wedge F_2 \wedge \ldots \wedge F_6 \to G)$

$(F_1 \wedge F_2 \ldots \wedge F_6 \wedge \neg G) \equiv \bot$ is want "$\bot$"

3. Encode and reason about the following.
If a scarcity of commodities develops, then the prices rise. If there is a change of government, then fiscal controls will not be continued. If the threat of inflation persists, then fiscal controls will be continued. If there is over-production, then prices do not rise. It has been found that there is over-production and there is a change of government. Therefore, neither the scarcity of commidities has developed, nor there is a threat of inflation.

$F_1: sc \rightarrow pr \equiv (\neg sc \vee pr)$

$F_2: cng \rightarrow \neg fc \equiv (\neg cng \vee \neg fc)$

$F_3: ti \rightarrow fc \equiv (\neg ti \vee fc)$

$F_4: ov \rightarrow \neg pr \equiv (\neg ov \vee \neg pr)$

$F_5: \overset{F_{51}}{ov} \wedge \overset{F_{52}}{cng}$

$\therefore G: (\underbrace{\neg sc}_{G_1} \wedge \underbrace{\neg ti}_{G_2\,?})$

$\neg G: (sc \vee ti)$

$(\underbrace{F_1 \wedge \cdots \wedge F_5 \wedge \neg G}_{} \equiv 1)$

To prove.

1. F_4, F_{51} / $\therefore \neg pr$

2. F_1, $\neg pr$ / $\therefore \underbrace{\neg sc}_{G_1}$

3. F_2, F_{52} / $\therefore \neg fc$

4. F_3, $\neg fc$ / $\therefore \underbrace{\neg ti}_{G_2}$

$\boxed{R\text{-}R}$

1. F_{51}, F_4 / $\neg pr$

2. F_1, $\neg pr$ / $\neg sc$

3. $\neg G$, $\neg sc$ / ti

4. ti, F_3 / fc

5. F_2, sc / $\neg cng$

6. $\neg cng$, F_{52} / $\therefore \bot$

4. Prove that, $\forall x\, [P(x) \rightarrow (Q(x) \leftrightarrow R(x))]$ is equivalent to
$[\, \forall x\, [(P(x) \wedge Q(x)) \rightarrow R(x)]\,] \wedge [\forall x\, [(P(x) \wedge R(x)) \rightarrow Q(x)]\,]$

$$(S_1 \leftrightarrow S_2) \equiv T \quad \text{(Tmy)} \qquad T_1 \qquad S_2 \xrightarrow{\text{Simplify}} S_1 \qquad T_2$$

$$T_1 \equiv \forall x \left[\neg P(x) \vee \neg Q(x) \vee R(x) \right] \quad \wedge$$

$$T_2 \equiv \forall x \left[\neg P(x) \vee \neg R(x) \vee Q(x) \right]$$

RHS $\equiv$

$$T_1 \wedge T_2 \equiv \forall x \left[\neg P(x) \vee \left\{ \underbrace{(\neg Q(x) \vee R(x))}_{Q(x) \rightarrow R(x)} \wedge \underbrace{(\neg R(x) \vee Q(x))}_{R(x) \rightarrow Q(x)} \right\} \right]$$

$$[\, Q(x) \leftrightarrow R(x) \,]$$

$$\equiv \forall x \left[P(x) \rightarrow (Q(x) \leftrightarrow R(x)) \right]$$

$\equiv$ LHS

5. Formalize the following sentences in first-order logic using only the follwoing predicates.

~~inside(x, y) : x is inside of y~~ ~~free(x) : x is free~~

love(x, y) : x loves y diff(x, y) : x differs from y

(a) There is at least one person who loves Mary.
(b) There is exactly one person who loves Mary. w
(c) There is at most one person who loves Mary.

? There are exactly two persons who loves Mary.

(a) $\exists x \; love(x, Mary)$

(b) $\exists x [love(x, Mary) \wedge \forall y (diff(x, y) \rightarrow \neg love(y, Mary))]$

$\forall y [love(y, Mary) \rightarrow \neg diff(x, y)]$

(c) $[b] \vee [\forall y \neg love(y, Mary)]$

6. Translate the following into idiomatic/concise English statement: ∀x [[H(x) ∧ ∀y¬M(x, y)] → U(x)], where H(x): x is a man, M(x, y): x is married to y, U(x): x is unhappy, and x and y range over people.

↳ All unmarried men are unhappy.

7. Encode the following logical statements using predicate logic (formulate suitable predicate and function symbols as required), and conclude on the validity of the last statement.

No man who is a candidate will be defeated if he is a good campaigner. Any man who runs for office is a candidate. Any candidate who is not defeated will be elected. Every man who is elected is a good campaigner. Therefore, Any man who runs for office will be elected if and only if he is a good campaigner.

$$F_1: \forall x \left[(cand(x) \land camp(x)) \rightarrow \neg def(x) \right]$$

$$F_2: \forall x \left[off(x) \rightarrow cand(x) \right]$$

$$F_3: \forall x \left[(cand(x) \land \neg def(x)) \rightarrow elec(x) \right]$$

$$F_4: \forall x \left[elec(x) \rightarrow camp(x) \right]$$

$cand(x)$
$def(x)$
$camp(x)$
$off(x)$
$elec(x)$

$$G: \forall x \left[off(x) \rightarrow (elec(x) \leftrightarrow camp(x)) \right] \equiv G_1 \land G_2$$

$$\neg G_1: \cancel{\forall} \exists x \, \neg\left[(off(x) \land elec(x)) \rightarrow camp(x) \right]$$

$$G_2: \forall x \left[(off(x) \land camp(x)) \rightarrow elec(x) \right]$$

$F_1: (\neg cand(x) \vee \neg camp(x) \vee \neg def(x))$ C_1

$G = G_1 \wedge G_2$

$F_2: (\neg off(x) \vee cand(x))$ C_2

$\therefore \neg G = (\neg G_1 \vee \neg G_2)$

$F_3: (\neg cand(x) \vee def(x) \vee elect(x))$ C_3

CNF

$C_1 \wedge C_2$ ---

$(x_1 \vee x_2 \vee x_3)$

$F_4: (\neg elec(x) \vee camp(x))$ C_4

$\neg G_2: \cancel{\exists x} [off(A) \wedge camp(A) \wedge \neg elec(A)]$

$\vee$

$\neg G_1: \cancel{\exists x} [off(A) \wedge elec(A) \wedge \neg camp(A)]$

G_{11} G_{12} G_{13}

F_1

x/A

G_{11}

. . - - - - - - .

Resolution Refut. $\longrightarrow \perp$