Tutorial 9
Space Complexity

1. Show that
   (a) **PSPACE** is closed under union, intersection, complement and Kleene Closure.
   (b) **NL** is closed under union, intersection and Kleene Closure.

2. A ladder is a sequence of strings $s_1, s_2, \ldots, s_k$, wherein every string differs from the preceding one in exactly one character. For example, the following is a ladder of English words, starting with "head" and ending with "free": head, hear, near, fear, bear, beer, deer, deed, feed, feet, fret, free. Let

$$\mathsf{LADDER}_{DFA} = \{\langle \mathcal{M}, s, t\rangle : \mathcal{M} \text{ is a DFA and } L(\mathcal{M}) \text{ consists of a ladder of strings starting with } s \text{ and ending with } t\},$$

   where $s, t \in \Sigma^*$ and $\mathcal{M}$ is defined over the input alphabet Σ. Show that LADDER_{DFA} is in **PSPACE**.

   Hint: Use the fact that **PSPACE** = **NPSPACE**.

3. In the generalised version of the game Tic-Tac-Toe, 2 players places marks X (crosses) and O (noughts) on an $m \times n$ grid. A player wins if she is the first to place k marks in a row, column or diagonal. The game ends in a draw if no such sequence is present when all the mn cells of the grid are filled. Assuming that X always starts, show that the language

$$\mathsf{GTICTACTOE} = \{\langle m, n, k, c\rangle : c \text{ is an intermediate configuration on the } m \times n \text{ board with next move by } X \text{ and } \exists \text{ a winning strategy for } X\}$$

   is in **PSPACE**.

4. Let $\mathbf{polyL} = \cup_{c>0}\mathbf{DSPACE}(\log^c n)$. Let **SC** (named after Stephen Cook) be the class of languages that can be decided by deterministic machines that run in polynomial time and $\log^c n$ space for some $c > 0$.
   (a) It is an open problem whether $\mathsf{PATH} \in \mathbf{SC}$. Why does Savitch's theorem not resolve this question?
   (b) Is $\mathbf{SC} = \mathbf{polyL} \cap \mathbf{P}$?

5. Show that 2SAT is in **NL**.