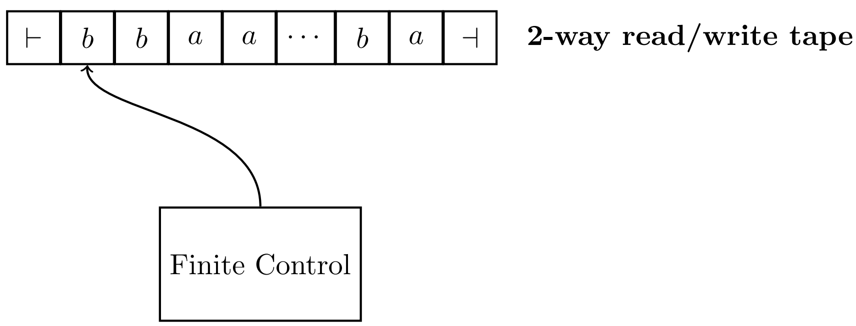
CS60005: Foundations of Computing Science — Autumn, 2021

# Tutorial 6

## Turing Machines, Recursive and R.E. Sets

1. Design a total Turing machine that accepts the set $\{0^{2^i} \mid i \geq 1\}$.

2. A linear bounded automaton (LBA) is exactly like a 1-tape TM, except that the input string $x \in \Sigma^*$ is enclosed in left and right endmarkers $\vdash$ and $\dashv$ which may not be overwritten. The machine is constrained never to move left of $\vdash$ or right of $\dashv$. It is allowed to read/write between these markers.




   (a) Give a rigorous formal definition of deterministic linearly bounded automata, including a definition of configurations and acceptance.

   (b) Let $\mathcal{M}$ be an LBA with state set $Q$ of size $k$ and tape alphabet $\Gamma$ of size $m$. How many possible configurations are there on input $x$ with $|x| = n$?

   (c) Argue that it is possible to detect in finite time whether an LBA loops on a given input.

   **Hint:** Use part (b).

3. Show that the class of recursively enumerable sets is closed under union and intersection.

4. Prove that a language $L$ is recursive if and only if there is an enumeration machine enumerating the strings of $L$ in a non-decreasing order of length (string of the same length may be arranged in lexicographic order). For example, strings of $0, 1^*$ would be arranged as $0, 1, 00, 01, 10, 11, 000, 001, 010, 011, 100, 101, 110, 111, \ldots$.

5. Is the set $\{\mathcal{M} \mid \mathcal{L}(\mathcal{M})$ contains atleast 100 elements$\}$ recursively enumerable.