CS60005 Foundations of Computing Science
Tutorial 5

Context-free Languages and Push-down Automata

1. What context-free languages will be generated by the following two (separate) context-free grammars, $G = \big(\{S, A, B\}, \{a, b, c\}, P, S\big)$, where P consists of the following production rules?

$$(a)\quad \begin{aligned} S &\rightarrow ASB \mid \varepsilon \\ A &\rightarrow a \\ B &\rightarrow bb \mid b \end{aligned} \qquad\qquad (b)\quad \begin{aligned} S &\rightarrow abScB \mid \varepsilon \\ B &\rightarrow bB \mid b \end{aligned}$$

2. Define the context-free grammars for the following context-free languages. Are your defined CFGs ambiguous / non-ambiguous?

   - $L_{2a} = \{a^i b^j c^k \mid i, j, k \geq 0,$ and $i = j$ or $i = k\}$
   - $L_{2b} = \{a^i b^j c^k \mid i, j, k \geq 0,$ and $i + j = k\}$
   - $L_{2c} = \{a^i b^j c^k \mid i, j, k \geq 0,$ and $i + k = j\}$
   - $L_{2d} = \{a^i b^j c^k \mid i, j, k \geq 0,$ and $i + k < j\}$
   - $L_{2e} = \{w \mid w \in a, b^*$ and $w \neq w^{rev}\} =$ all non-palindromes over $\{a, b\}$

   Now, design (separate) push-down automata which accepts all of the above CFL. Comment whether your PDA accepts by final state or empty stack.

3. Which of the following language(s) is/are context-free? Give justifications – For CFL, you need to give the CFG, otherwise you have to prove using Pumping Lemma for CFL.

   - $L_{3a} = \big\{a^m b^n \mid m, n \geq 0, m = 2n\big\}$
   - $L_{3b} = \big\{a^m b^n \mid m, n \geq 0, m \neq 2n\big\}$
   - $L_{3c} = \big\{a^m b^n \mid n \geq 0, 3n \leq m \leq 5n\big\}$
   - $L_{3d} = \big\{a^m b^n c^{m+n} \mid m, n \geq 1\big\}$
   - $L_{3e} = \big\{a^m b^m c^{m+n} \mid m, n \geq 1\big\}$
   - $L_{3f} = \big\{a^l b^m c^n \mid l \geq 0, l < m$ and $l < n\big\}$

4. Convert the following context-free grammars into equivalent CFG in Chomsky Normal Form. Then, proceed converting these Chomsky Normal Form CFGs into Greibach Normal Form CFGs.

$$(a)\quad \begin{aligned} S &\rightarrow BSB \mid B \mid \varepsilon \\ B &\rightarrow 00 \mid \varepsilon \end{aligned} \qquad (b)\quad \begin{aligned} S &\rightarrow ABa \mid \varepsilon \\ A &\rightarrow aab \\ B &\rightarrow Ac \end{aligned} \qquad (c)\quad \begin{aligned} S &\rightarrow AbA \\ A &\rightarrow Aa \mid \varepsilon \end{aligned}$$

5. Given the following languages over the alphabet $\{a, b\}$, design one-state PDAs that accepts by empty stack (separate PDA for each one).

   - $L_{5a} = (a + b)^* b$
   - $L_{5b} = a(a + b)^* b$
   - $L_{5c} =$ all palindromes over $\{a, b\}$

   Explore the following:

   (a) Since L_{5a} and L_{5b} are regular languages, can you directly present the left linear and the right linear grammar for them and then formally derive the NFAs from these grammars?

   (b) Can you also develop two (separate) DFAs for L_{5a} and L_{5b} directly? Then, from those DFAs, can you again formally derive the grammars for the same?

6. Prove that, the following context-free grammar, $G = \big(\{S\}, \{a, b, c\}, P, S\big)$, is ambiguous. Here, the production rules (P) are given as: $S \rightarrow aS \mid aSbS \mid c$.

   Construct a non-ambiguous grammar, G', that derives the same language. Also prove $\mathcal{L}(G) = \mathcal{L}(G')$.

7. Consider the two CFGs G and G' with the start symbols S and S' and with the only productions:

   Productions of G : $S \rightarrow aS \mid B, \quad B \rightarrow bB \mid b$

   Productions of G' : $S' \rightarrow aA' \mid bB', \quad A' \rightarrow aA' \mid B', \quad B' \rightarrow bB' \mid \varepsilon$

   Prove that, $\mathcal{L}(G) \subset \mathcal{L}(G')$, i.e., $\mathcal{L}(G)$ is strictly contained in $\mathcal{L}(G')$.