CS60005 Foundations of Computing Science
Tutorial 4

Finite Automata and Regular Languages

1. Consider the following language over the alphabet $\Sigma = \{a, b\}$:

$$L_1 = \Big\{x \in \{a,b\}^* \mid x \text{ starts with } ab \text{ but does not end with } ab\Big\}.$$

   (a) Write a regular expression for L_1.

   (b) Design a DFA for L_1.

2. Construct a regular expression over the alphabet $\Sigma = \{a, b, c\}$ for:

$$L_2 = \Big\{x \in \{a,b,c\}^* \mid x \text{ has } 4i+1 \text{ } b\text{'s for some integer } i \geq 0\Big\}.$$

3. The language $L_4 = \big\{uvv^rw \mid u, v, w \in \{a,b\}^+\big\}$ is regular. Here v^r is the reverse of v. Answer the following questions.

   (a) Design a regular expression whose language is L_4.

   (b) Convert the regular expression of Part-(a) to an equivalent NFA.

   (c) Convert the NFA in Part-(b) into an equivalent DFA using *subset-construction procedure*.

   (d) Optimize the number of states of the DFA obtained in Part-(c) to produce the minimum state DFA.

4. Use pumping lemma to prove that the following languages are non-regular:

   - $L_{4a} = \big\{a^{k^3} \mid k \geq 0\big\}$.
   - $L_{4b} = \big\{a^{n!} \mid n \geq 0\big\}$.
   - $L_{4c} = \big\{a^p \mid p \text{ is prime}\big\}$.
   - $L_{4d} = \big\{a^ib^ja^{ij} \mid i, j \geq 0\big\}$.

5. Determine the regularity/non-regularity of the following languages:

   - $L_{5a} = \Big\{x \in \{a,b\}^* \mid \#a(x) - \#b(x) = 2021\Big\}$.
   - $L_{5b} = \Big\{x \in \{a,b\}^* \mid \#a(x) - \#b(x) \text{ is a multiple of } 2021\Big\}$.

   where, $\#a(x)$ and $\#b(x)$ denote the number of a's and b's in x.

   [*Hint:* Instead of finding regular expressions or using pumping lemma, you may alternatively try using the MyhillNerode theorem and show whether the number of equivalent classes formed is finite/infinite.]

6. Let A, B be languages over an alphabet Σ, and $C = A - B$. Then, justify which of the following statements is/are true?

   (a) If A and B are regular, then C is regular.

   (b) If A and C are regular, then B is regular.

   (c) If B and C are regular, then A is regular.

   (d) If C is regular, then A and B are regular.

7. Two regular expressions over the same alphabet are called *equivalent* if they generate the same language. Prove/Disprove the equivalence of the following pairs of regular expressions over $\{a, b\}$.

   (a) $(ab + a)^*a$ and $a(ba + a)^*$.

   (b) $(ab^*a + ba^*b)^*$ and $(ab^*a)^* + (ba^*b)^*$.