Indian Institute of Technology Kharagpur
Department of Computer Science and Engineering

Foundations of Computing Science (CS60005) **Autumn Semester, 2021-2022**

Test 3 [Marks = 30] **Date: 17-Nov-2021, 08:15 – 09:30** **Venue: Online**

There are ALL THREE questions. State all assumptions you make. Be brief and precise.

1. (a) Prove or disprove: $\{\mathcal{M} \mid \mathcal{M} \text{ runs in time } O(n^3)\}$ is undecidable. [6]

   (b) For a set $A \subseteq \Sigma^*$ (with $|\Sigma| \geq 2$), define $A^{\mathbf{R}} = \{w^{\mathbf{R}} \mid w \in A\}$ where $w^{\mathbf{R}}$ denotes w reversed. Is it decidable for a given TM $\mathcal{M}$ whether $L(\mathcal{M}) = L(\mathcal{M})^{\mathbf{R}}$? Justify your answer. [4]

2. Answer true or false. Justify your answer.

   (a) All **NP**-Hard problems are decidable. [2]

   (b) If L_1, L_2 are **NP**-Complete, then so is $L_1 \cup L_2$. [3]

   (c) If every **NP**-hard language is **PSPACE**-hard, then **PSPACE** = **NP**. [2]

   (d) **polyL** $\neq$ **polyNL**,
   where **polyL** $= \cup_{c>0}$**DSPACE**$(\log^c n)$ and **polyNL** $= \cup_{c>0}$**NSPACE**$(\log^c n)$. [3]

3. (a) Consider a variant M-3SAT (*minimal* 3SAT) of 3SAT defined as follows: a Boolean formula $\phi \in$ M-3SAT if there exists atleast one satisfying assignment for ϕ where exactly one literal in every clause is true. Show that M-3SAT is **NP**-complete. [5]

   (b) Let A, B be two $n \times n$ Boolean matrices. Entries of the *Boolean product* $C = A \cdot B$ are defined as $C_{ij} = \vee_{k=1}^{n}(a_{ik} \wedge b_{kj})$ for $1 \leq i, j \leq n$. Describe a logspace algorithm to compute $A \cdot B$ given A, B. [5]