Indian Institute of Technology Kharagpur
Department of Computer Science and Engineering

Foundations of Computing Science (CS60005) **Autumn Semester, 2021-2022**

Test - 2 [Marks: 30] **Date: 06-Oct-2021 (Wednesday), 8:15am – 9:30am** **Venue: Online**

[Instructions: ***There are THREE questions. Answer ALL questions. Be brief and precise.*** **]**

Q1. One of the following languages over the alphabet $\{a, b\}$ is regular; the other one is not regular. Identify which one is what and Justify. *No credit will be given only for correct identification (without justification provided).*

$(\mathbf{a})\quad L_1 = \left\{\alpha\beta\alpha \mid \alpha \in \{a,b\}^+ \text{ and } \beta \in \{a,b\}^+\right\}$ $(\mathbf{b})\quad L_2 = \left\{\alpha\beta\alpha \mid \alpha \in \{a\}^+ \text{ and } \beta \in \{a,b\}^+\right\}$

In particular, do the following: (i) For the one which is *regular*, give the corresponding Regular Expression as well as the minimum-state deterministic finite automaton (DFA) *both*; and (ii) For the one which is *not regular*, give a proof using the Pumping Lemma.

[Marks: (3 + 3) + 4 = 10]

Q2. Let $\mathcal{B}_n$ $(n \geq 1)$ denote the binary representation (string with leading zeros omitted) corresponding to any $n \in \mathbb{N}$. For example, you may consider, $\mathcal{B}_5 = 101$, $\mathcal{B}_{10} = 1010$, and $\mathcal{B}_{13} = 1101$. Let $\#$ be another symbol not in $\{0, 1\}$. We denote $\mathcal{B}_n^{rev}$ to indicate the reverse of string $\mathcal{B}_n$. For example, $\mathcal{B}_5^{rev} = 101$, $\mathcal{B}_{10}^{rev} = 0101$, and $\mathcal{B}_{13}^{rev} = 1011$.

One of the following languages over the alphabet $\{0, 1, \#\}$ is context-free; the other one is not context-free. Identify which one is what and Justify. *No credit will be given only for correct identification (without justification provided).*

$(\mathbf{a})\quad L_3 = \left\{\mathcal{B}_n \# \mathcal{B}_{n+1} \mid n \geq 1\right\}$ $(\mathbf{b})\quad L_4 = \left\{\mathcal{B}_n^{rev} \# \mathcal{B}_{n+1} \mid n \geq 1\right\}$

In particular, do the following: (i) For the one which is *a context-free language (CFL)*, give the corresponding context-free grammar (CFG) as well as the push-down automaton (PDA) *both*; and (ii) For the one which is *not a CFL*, give a proof using the Pumping Lemma for CFL.

[Marks: (3 + 3) + 4 = 10]

Q3. Consider the language $E = \{w \in \{0,1\}^* \mid$ number of 0's in w is twice the number of 1's$\}$.

(a) Design a total single tape Turing machine that accepts this language. Precisely write down the tape alphabet, the set of states and the transition function.

(b) Write down the sequence of configurations of the Turing machine from part (a) on the input strings 101100 and 010010.

[Marks: 8 + (1 + 1) = 10]