Indian Institute of Technology Kharagpur
Department of Computer Science and Engineering

Foundations of Computing Science (CS60005) **Autumn Semester, 2021-2022**

Test - 1 [Marks: 30] **Date: 08-Sep-2021 (Wednesday), 8:15am – 9:30am** **Venue: Online**

[Instructions: *There are FOUR questions. Answer ALL questions. Be brief and precise.*]

Q1. You are about to leave for university classes in the morning and discover you do not have your glasses. You know that the following *six* statements are true:

F_1 : *If my glasses are on the kitchen table, then I saw them at breakfast.*

F_2 : *I was reading the newspaper in the living room or I was reading the newspaper in the kitchen.*

F_3 : *If I was reading the newspaper in the living room, then my glasses are on the coffee table.*

F_4 : *I did not see my glasses at breakfast.*

F_5 : *If I was reading my book in bed, then my glasses are on the bed table.*

F_6 : *If I was reading the newspaper in the kitchen, then my glasses are on the kitchen table.*

You task is to derive the answer to the following question logically – *"Where are the glasses?"*

Please frame logical arguments to formally deduce (applying logical inferencing) the answer to the above question. Present your solution as indicated in the following parts.

(a) Write all the propositions (that you have used) with English statements (meaning). **(1)**

(b) Build suitable propositional logic formula to encode each of the *six* statements $F_1 - F_6$ given above. **(3)**

(c) Use logical inferencing rules (or resolution-refutation principle) to completely derive the answer and conclude where do you find the glasses. **(3)**

Q2. Consider the following statements.

F_1 : *Tony and Mike are members of the Alpine club.*

F_2 : *Every member of the Alpine club is either a skier, or a mountain climber, or both.*

F_3 : *No mountain climber likes rain.*

F_4 : *All skier likes snow.*

F_5 : *Mike dislikes whatever Tony likes and likes whatever Tony dislikes.*

F_6 : *Tony likes rain and snow.*

Your tasks are to do the following:

(a) Write all the predicates (that you have used) with English statements (meaning). **(1)**

(b) Encode the above *six* statements $F_1 - F_6$ in predicate (first-order) logic. **(3)**

(c) Use resolution-refutation principle (logical deduction procedure) to prove that,
G : *"There is a member in the Alpine club who is a mountain climber, but not skier."* **(4)**

Q3. A partial order ρ on a set A is called a total order (or a linear order) if for any two different $a, b \in A$ either $a \ \rho \ b$ or $b \ \rho \ a$. Which of the following relations $\rho,\ \sigma,\ \tau$ on $\mathbb{N}$ (the set of natural numbers) are partial orders and/or total orders? – *Provide proper reasoning / justification*.

[Hint: For each of the relations, $\rho,\ \sigma,\ \tau$ on $\mathbb{N}$, first determine whether the relation is a partial order, and if so, then determine whether it is a total order.]

(a) $a \ \rho \ b$ if and only if $a \leq b + 1701$. **(2)**

(b) $a \ \sigma \ b$ if and only if a divides b, i.e. $b = ax$, for some $x \in \mathbb{N}$. **(3)**

(c) $a \ \tau \ b$ if and only if either $u < v$, or $u = v$ and $x \leq y$, where $a = 2^u x$ and $b = 2^v y$ with x and y odd. **(3)**

Q4. Prove or disprove the following *with proper reasoning / justification*.

(a) Let G be a multiplicative group in which $(ab)^{-1} = a^{-1}b^{-1}$ for all $a, b \in G$.
Then, prove or disprove that G is Abelian. **(2)**

(b) Let R be a ring. Two elements $a, b \in R$ are called associates, denoted $a \sim b$, if $a = ub$ for some unit u of R.
Then, prove or disprove that $\sim$ is an equivalence relation on R. **(3)**

(c) Prove or disprove that the set of all finite subsets of $\mathbb{N}$ (the set of natural numbers) is countable. **(2)**