Assignment 2

CS60050: Foundations of Computing Science — Autumn, 2021
Deadline: 8th November, 23:59 — Total Marks: 20

Solve all problems. Stick to notation used in the classes.
Write solutions on white paper, scan and then upload a single pdf file. Make sure that the file size does not exceed 20 MB. Any format other than pdf is not acceptable.
Upload in CSE-Moodle course page (suitable entry is already created)

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1. A *one-counter automaton* is an automaton with a finite set of states Q, a two-way read-only input head and a separate counter that can hold any non-negative integer. The input x is enclosed in endmarkers $\vdash, \dashv \notin \Sigma$ and the input head may not go outside the endmarkers. The machine starts in its start state s with its counter set to 0 and with its input head pointing to $\vdash$. In each step, it can test its counter for 0. Based on this information, its current state and the symbol its input head is currently reading, it can either add $1, -1$ to its counter and move its input head either left or right and enter a new state. It accepts by entering a distinguished final state t.
   - (a) Give a rigorous formal definition of these machines, including a definition of acceptance. Your definition should begin as follows: "A one-counter automaton is a 7-tuple $\mathcal{M} = (Q, \Sigma, \vdash, \dashv, s, t, \delta)$, where ...". $\boxed{2}$
   - (b) Prove that the membership problem (given $\mathcal{M}, x$, does $\mathcal{M}$ accept x?) for deterministic one-counter automata is decidable. $\boxed{2}$

2. Describe a language over alphabet 0 for each of the following classes and justify.
   - (a) Regular
   - (b) Recursive but not context-free
   - (c) Recursively enumberable but not recursive

   $\boxed{5 = (1+2+2)}$

3. Let L be the set of Turing machines $\mathcal{M}$ with input alphabet Σ such that $\mathcal{M}$ writes the symbol $a \in \Sigma$ at some point on its tape. Show that L is undecidable. $\boxed{3}$

4. Suppose that $\mathbf{P} \neq \mathbf{NP}$. Prove that it is undecidable, given $L \in \mathbf{NP}$, whether or not $L \in \mathbf{P}$. $\boxed{3}$

5. A language L is in class **DP** (where **D** stands for difference) iff there are languages $L_1 \in \mathbf{NP}$ and $L_2 \in \mathbf{coNP}$ so that $L = L_1 \cap L_2$.
   - (a) Define completeness for the class **DP** under polynomial time reductions. $\boxed{0.5}$
   - (b) The problem SAT-UNSAT is defined as the set of all pairs of Boolean formulae $\langle \phi, \psi \rangle$ such that ϕ is satisfiable and ψ is unsatisfiable. Show that SAT-UNSAT is **DP**-complete. $\boxed{2.5}$
   - (c) A (undirected) graph G is *Hamiltonian* if it contains a Hamiltonian cycle (a cycle visiting every vertex exactly once). The language HC-CRITICAL consists of all graphs G such that G is not Hamiltonian but adding any edge to G will make it Hamiltonian. Show that HC-CRITICAL is in **DP**. $\boxed{2}$