COMPLEXITY THEORY

Efficient Computation

Resources used during computation : space, time, randomness, ...

Quantify the amount of each resource

Efficiency of computation w.r.t. to a resource

$\equiv$ amount of resource is "small".

Model : Turing machines.

TIME COMPLEXITY

Running time

$f : \mathbb{N} \to \mathbb{N}$ $\qquad T : \mathbb{N} \to \mathbb{N}$

Turing machine $\quad$ M computes f if $\forall x \in \mathbb{N}$, M on input x, halts with $f(x)$ written on the tape.

M computes f in $T(n)$-time if M's computation on every input x requires $\leq T(|x|)$ steps.

Big-Oh

$f, g : \mathbb{N} \to \mathbb{N}$ $\qquad f(n) = O(g(n))$

$\exists c > 0, n_0 > 0$ s.t. $\forall n \geq n_0$, $f(n) \leq c \cdot g(n)$.

Small-Oh

$f(n) = o(g(n))$ if $\lim_{n \to \infty} \frac{f(n)}{g(n)} = 0$

Computation using TM Variants

$f: \mathbb{N} \to \{0,1\}$, $T: \mathbb{N} \to \mathbb{N}$

→ If f is $T(n)$-computable over alphabet Γ then f is $O(\log|\Gamma| \cdot T(n))$-computable over $\{0,1\}$

→ If f is computable in $T(n)$-time by a k-tape TM then f is computable in $O(k \cdot T(n)^2)$-time by a single tape TM.

→ If f is computable in $T(n)$ time by a 2-way infinite tape TM then it is computable in $O(T(n))$-time by a semi-infinite tape TM.

All deterministic variants are <u>polynomially</u> equivalent.

COMPLEXITY CLASS : set of functions / languages that can be computed / recognised within given resource bounds.

Time Complexity Classes

DTIME$(T(n))$: A language L is in DTIME$(T(n))$ iff L is decidable by a deterministic TM running in time $O(T(n))$.

$$P = \bigcup_{c \geq 1} \text{DTIME}(n^c)$$

$\downarrow$ captures our intuition of efficient computation (feasible)