Rice's theorem Part II : No non-monotone property of r.e. sets is semi-decidable (r.e.)

Proof: $\mathcal{P}$ is non-monotone & hence $\exists\ A = L(M_0)$, $B = L(M_1)$ s.t. $A \subseteq B$ & $\mathcal{P}(A) = T$ & $\mathcal{P}(B) = F$.

$$T_{\mathcal{P}} = \{ M \mid \mathcal{P}(L(M)) = T \}$$

$$\neg HP \le_m T_{\mathcal{P}}$$

$$(M, x) \longmapsto N$$

N: on i/p y
- run M_0 on y, M_1 on y & M on x simultaneously
- accept if
  M_0 accepts y
  OR
  M_1 accepts y & M halts on x.

$(M, x) \in \neg HP$ i.e., M does not halt on x

$\Rightarrow L(N) = L(M_0) = A$ $\quad \mathcal{P}(L(N)) = T$

$(M, x) \notin \neg HP$ i.e., M halts on x

$\Rightarrow L(N) = L(M_0) \cup L(M_1) = A \cup B = B$

$\mathcal{P}(L(N)) = F$

$\neg HP$ is not r.e. $\Rightarrow T_{\mathcal{P}}$ is not r.e., i.e, $\mathcal{P}$ is not semi-decidable