7. Enumeration Machines

- Consists of a finite control & 2 tapes –
  a read/write tape & a write-only output tape.
  2 tape-heads, one for each tape.
- Receives no input.

  similar to a TM
- Makes its moves according to a transition function
- Consists of a distinguished "enumeration state"
- When the machine enters enumeration state, the string written on the output tape is said to be "enumerated".
- Upon leaving enumeration state, the o/p tape is erased; o/p tape head goes to the left end; and the machine continues.
- The machine runs forever.

Language of an enumeration machine is the set of all strings ever enumerated by the machine.

Enumeration machines are equivalent to TMs

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1. If $A = L(E)$ for some enumeration machine E, then $A = L(M)$ for some TM M.

M : on input x

$x \in \Sigma^*$ where Σ is the alphabet of E.

(i) Run E

(ii) Everytime E enumerates a string, compare it with x.
If there is a match, then accept & halt.
Otherwise continue to run E.

$x \in L(M)$ iff $x \in L(E)$. $\therefore$ $A = L(M)$.

2. If $A = L(M)$ for some Turing machine M, then $A = L(E)$ for an enumeration machine E.

E:

- Let $x_1, x_2, x_3, \ldots$ be an ordering of all possible input strings for M. (i.e., an ordering of strings in Σ^*)

$\times$
- for $i = 1, 2, 3, \ldots$
  run M on x_i
  if M accepts, then enumerate x_i

Suppose $x_2 \notin L(M)$ & M loops on x_2 then E will never enumerate anything beyond x_2

- for $i = 1, 2, 3, \ldots$
  run M on $x_1, x_2, \ldots, x_i$ for 1 step
  if M accepts some x_j, then enumerate x_j.

$x \in L(E)$ iff $x \in L(M)$.

PROPERTIES OF RECURSIVE AND R.E. SETS

→ Recursive sets $\subsetneq$ R.E. sets

E.g.: Membership Problem

$MP = \{(M, x) : M \text{ is a TM accepting string } x\}$

Halting Problem

$HP = \{(M, x) : M \text{ is a TM halts on i/p } x\}$

→ Recursive sets are closed under complement.

A is recursive $\Rightarrow \neg A$ is recursive.

A is recursive $\Rightarrow \exists$ a total TM M accepting A.

N : on i/p x
- runs M on x
- accepts if M rejects
- rejects otherwise.

$L(N) = \{x : x \notin L(M)\}$

$= \neg A$

3. R.E. sets are not closed under complement.

co-R.E. : set of languages whose complements are in R.E.

4. A & $\neg A$ are both r.e. $\Rightarrow$ A is recursive.

A is r.e. $\Rightarrow \exists M$ accepting A

$\neg A$ is r.e. $\Rightarrow \exists N$ accepting $\neg A$

K: on i/p x

- run M on x & N on x
- if M accepts, then accept
- if N accepts, then reject.

K: total since for any x, either M or N accepts.

$L(K) = A \Rightarrow A$ is recursive.

Decidability & Semi-Decidability

P : property of strings

P is decidable $\iff \{x : P(x) = T\}$ is recursive

P is semi-decidable $\iff \{x : P(x) = T\}$ is r.e.

A is recursive $\iff$ property "$x \in A$" is decidable

A is r.e. $\iff$ property "$x \in A$" is semi-decidable.